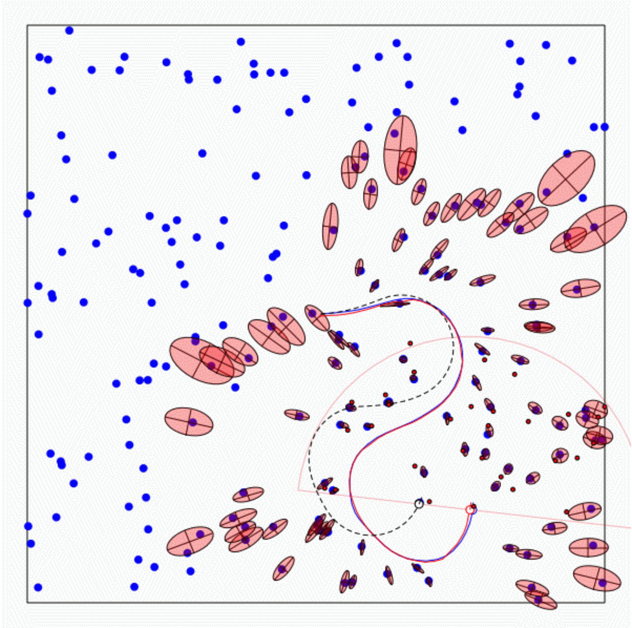
Probabilistic Robotics

SLAM and FastSLAM

(lightly modified version of the slideset accompanying the Thrun, Burgard, Fox book)

The SLAM Problem



A robot is exploring an unknown, static environment.

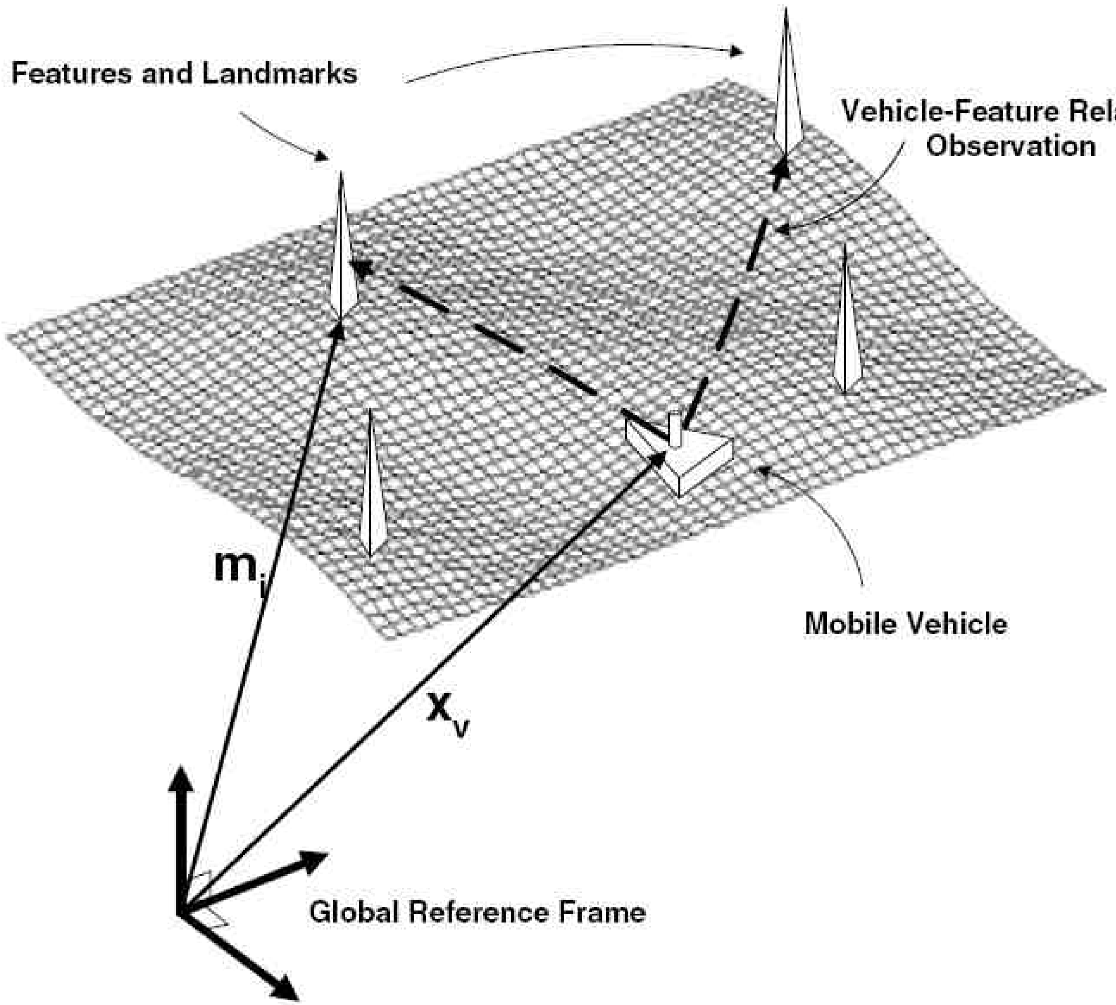
Given:

- The robot's controls
- Observations of nearby features

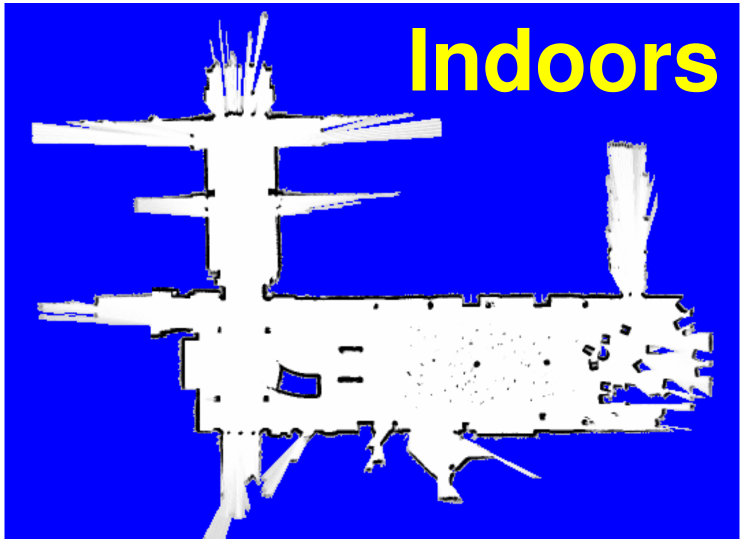
Estimate:

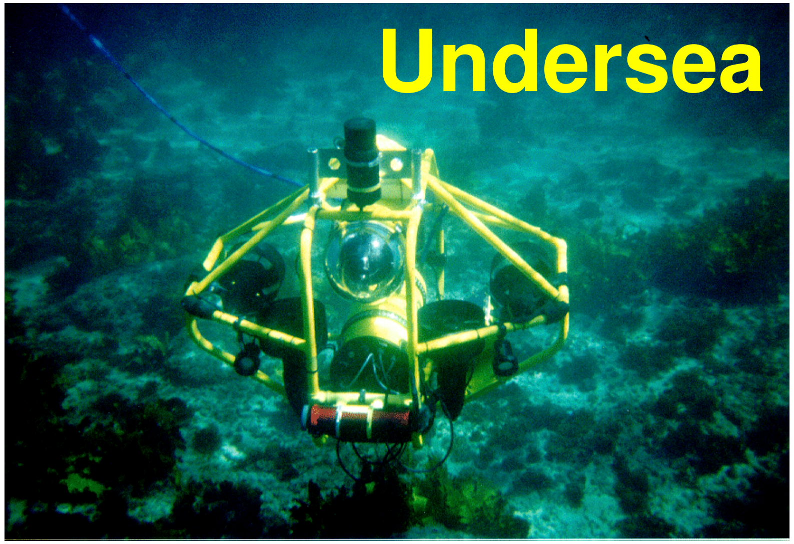
- Map of features
- Path of the robot

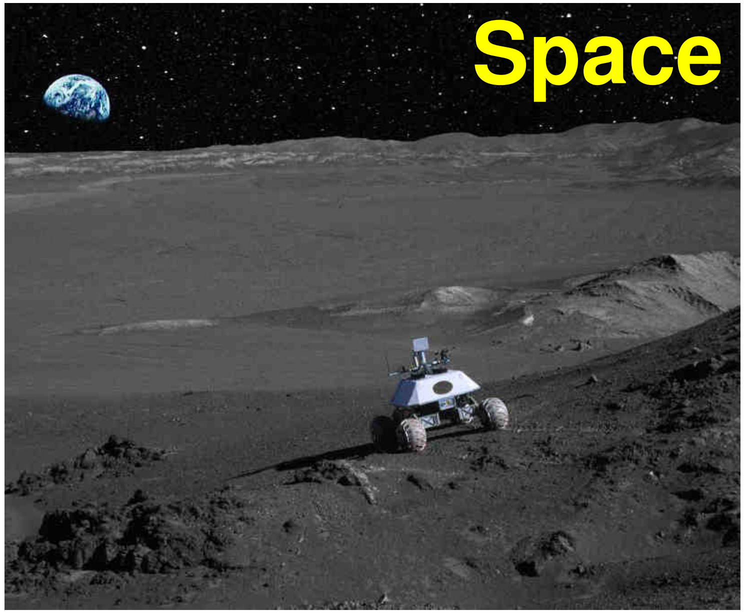
Structure of the Landmark-based SLAM-Problem



SLAM Applications

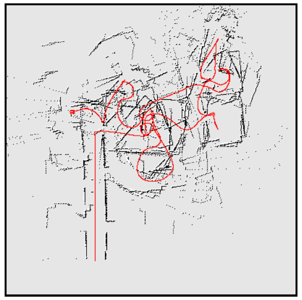
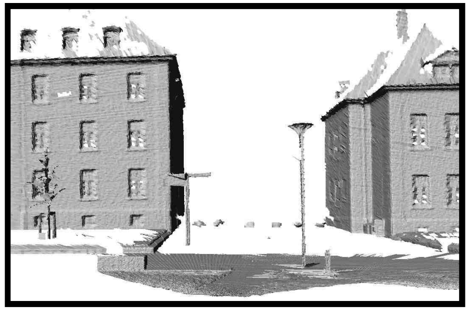
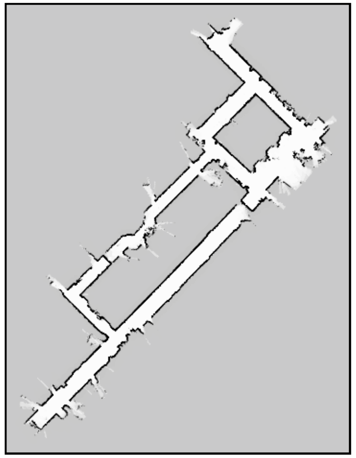






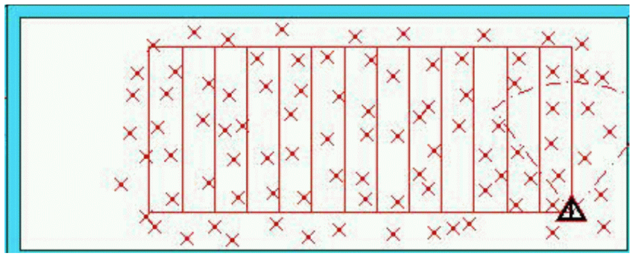
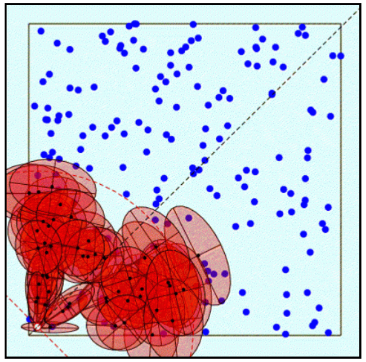
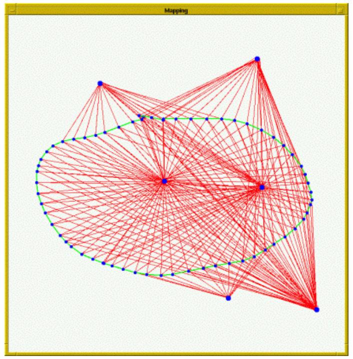
Representations

- Grid maps or scans

[Lu & Milios, 97; Gutmann, 98: Thrun 98; Burgard, 99; Konolige & Gutmann, 00; Thrun, 00; Arras, 99; Haehnel, 01;…]

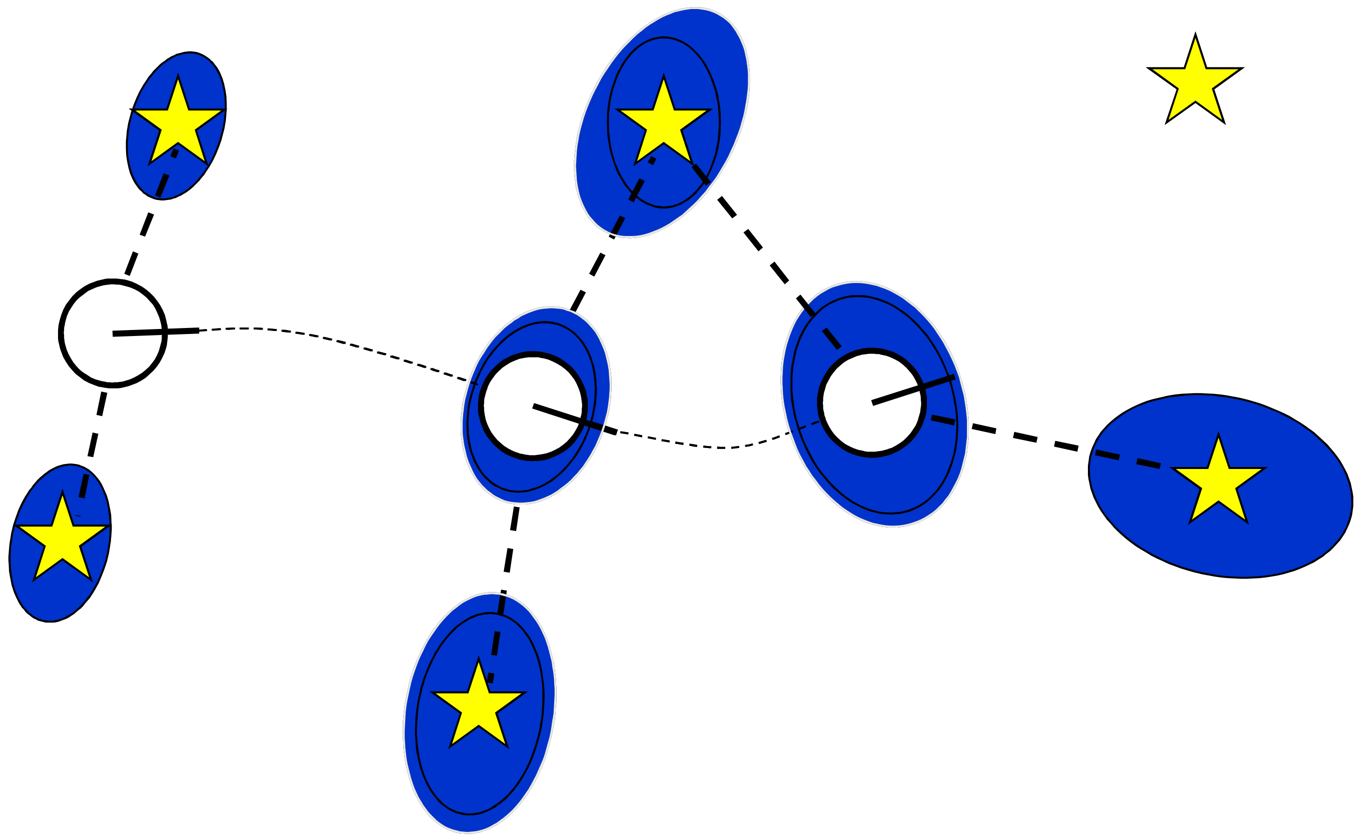
- Landmark-based

[Leonard et al., 98; Castelanos et al., 99: Dissanayake et al., 2001; Montemerlo et al., 2002;…

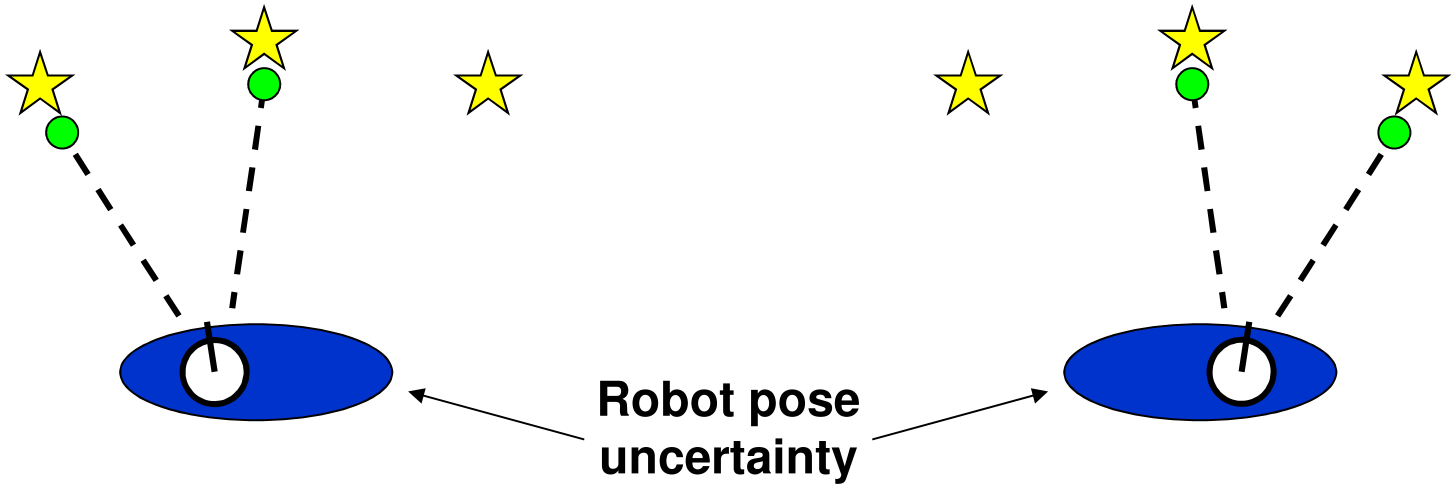
Why is SLAM a hard problem?

SLAM: robot path and map are both **unknown**



Robot path error correlates errors in the map

Why is SLAM a hard problem?



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
- Pose error correlates data associations

SLAM:
Simultaneous Localization and Mapping

- Full SLAM: Estimates entire path and map!

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

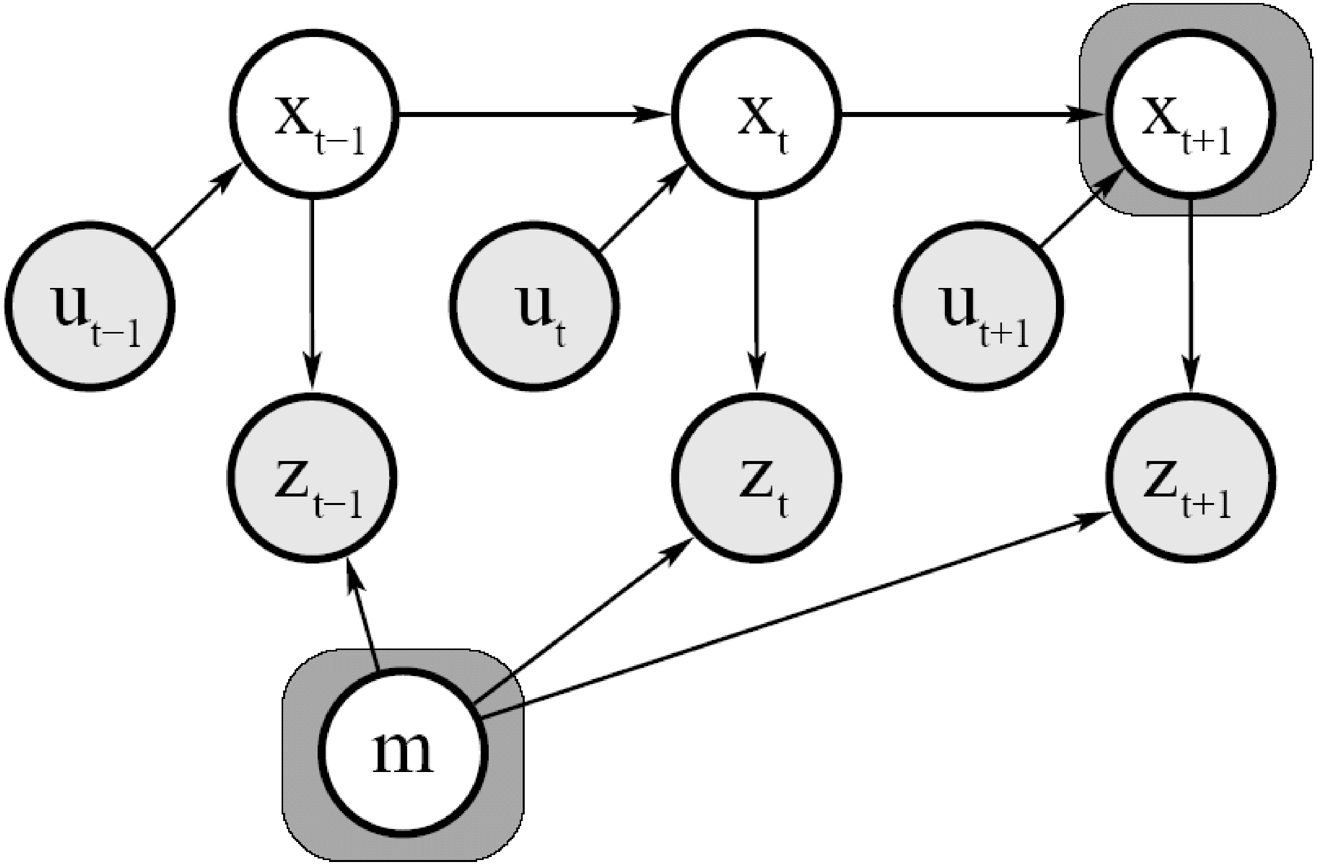
- Online SLAM:

$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int\int \ldots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t})\, dx_1 dx_2 \ldots dx_{t-1}$$

Integrations typically done one at a time

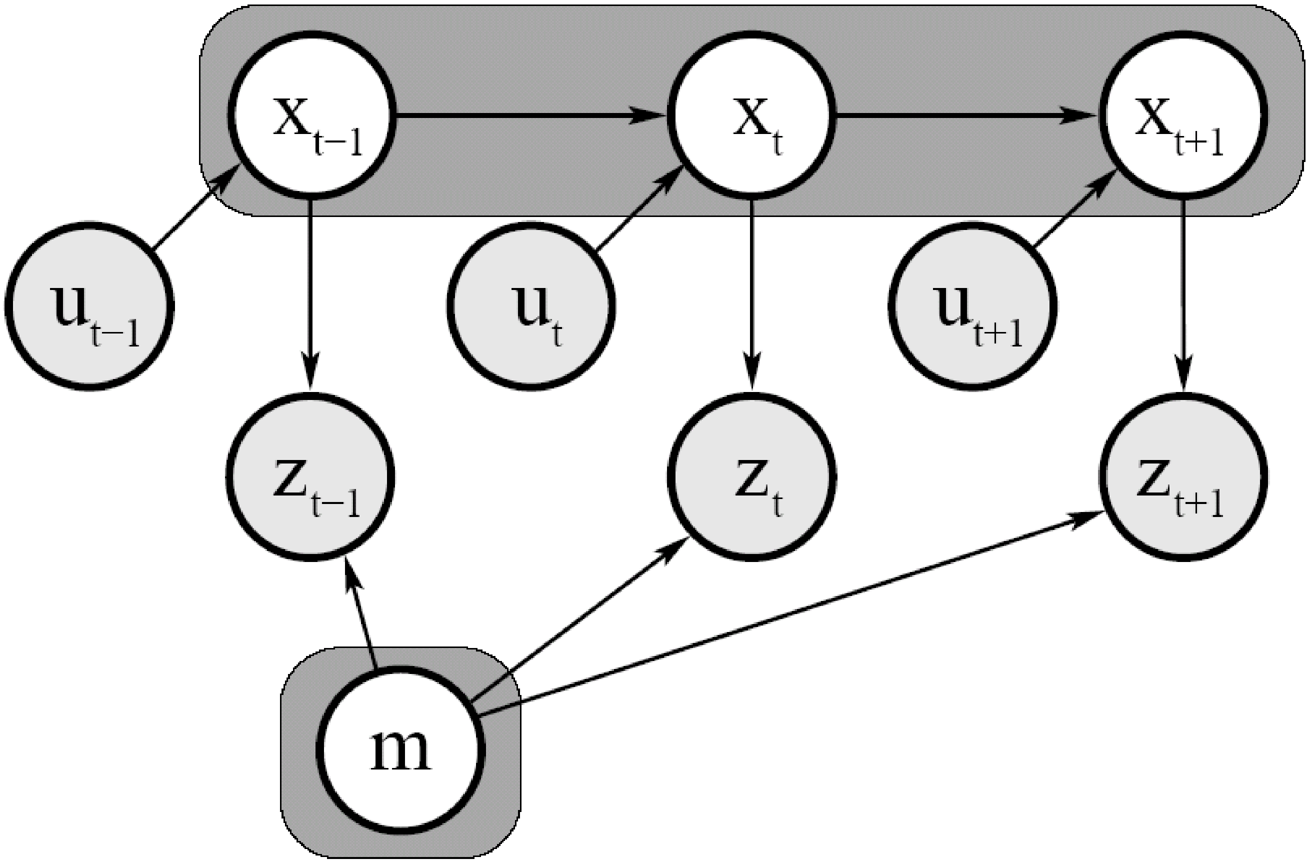
Estimates most recent pose and map!

Graphical Model of Online SLAM:



$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int \int \ldots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \, dx_1 dx_2 \ldots dx_{t-1}$$

Graphical Model of Full SLAM:



$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

Techniques for Generating Consistent Maps

- Scan matching
- EKF SLAM
- Fast-SLAM
- Probabilistic mapping with a single map and a posterior about poses
  Mapping + Localization
- Graph-SLAM, SEIFs

Scan Matching

Maximize the likelihood of the i-th pose and map relative to the (i-1)-th pose and map.

$$\hat{x}_t = \underset{x_t}{\operatorname{argmax}} \left\{ p(z_t \mid x_t, \hat{m}^{[t-1]}) \cdot p(x_t \mid u_{t-1}, \hat{x}_{t-1}) \right\}$$

current measurement

robot motion

map constructed so far

Calculate the map $\hat{m}^{[t]}$ according to “mapping with known poses” based on the poses and observations.

Kalman Filter Algorithm

1. Algorithm **Kalman_filter**(μ_{t-1}, Σ_{t-1}, u_t, z_t):

2. Prediction:
3. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:
6. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

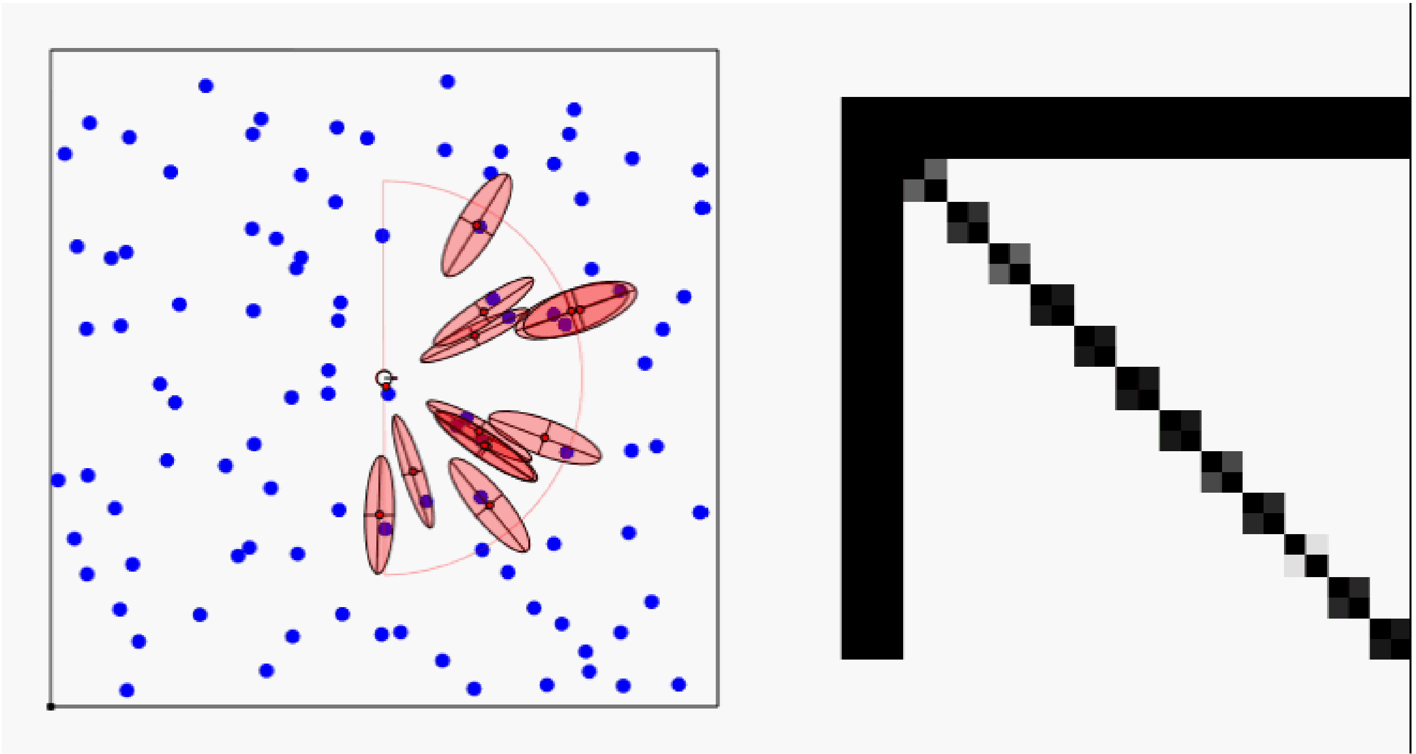
(E)KF-SLAM

- Map with N landmarks:(3+2N)-dimensional Gaussian

$$Bel(x_t, m_t) = \left\langle \begin{pmatrix} x \\ y \\ \theta \\ l_1 \\ l_2 \\ \vdots \\ l_N \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xl_1} & \sigma_{xl_2} & \cdots & \sigma_{xl_N} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} & \sigma_{yl_1} & \sigma_{yl_2} & \cdots & \sigma_{yl_N} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 & \sigma_{\theta l_1} & \sigma_{\theta l_2} & \cdots & \sigma_{\theta l_N} \\ \sigma_{xl_1} & \sigma_{yl_1} & \sigma_{\theta l_1} & \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} \\ \sigma_{xl_2} & \sigma_{yl_2} & \sigma_{\theta l_2} & \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{xl_N} & \sigma_{yl_N} & \sigma_{\theta l_N} & \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 \end{pmatrix} \right\rangle$$

- Can handle hundreds of dimensions

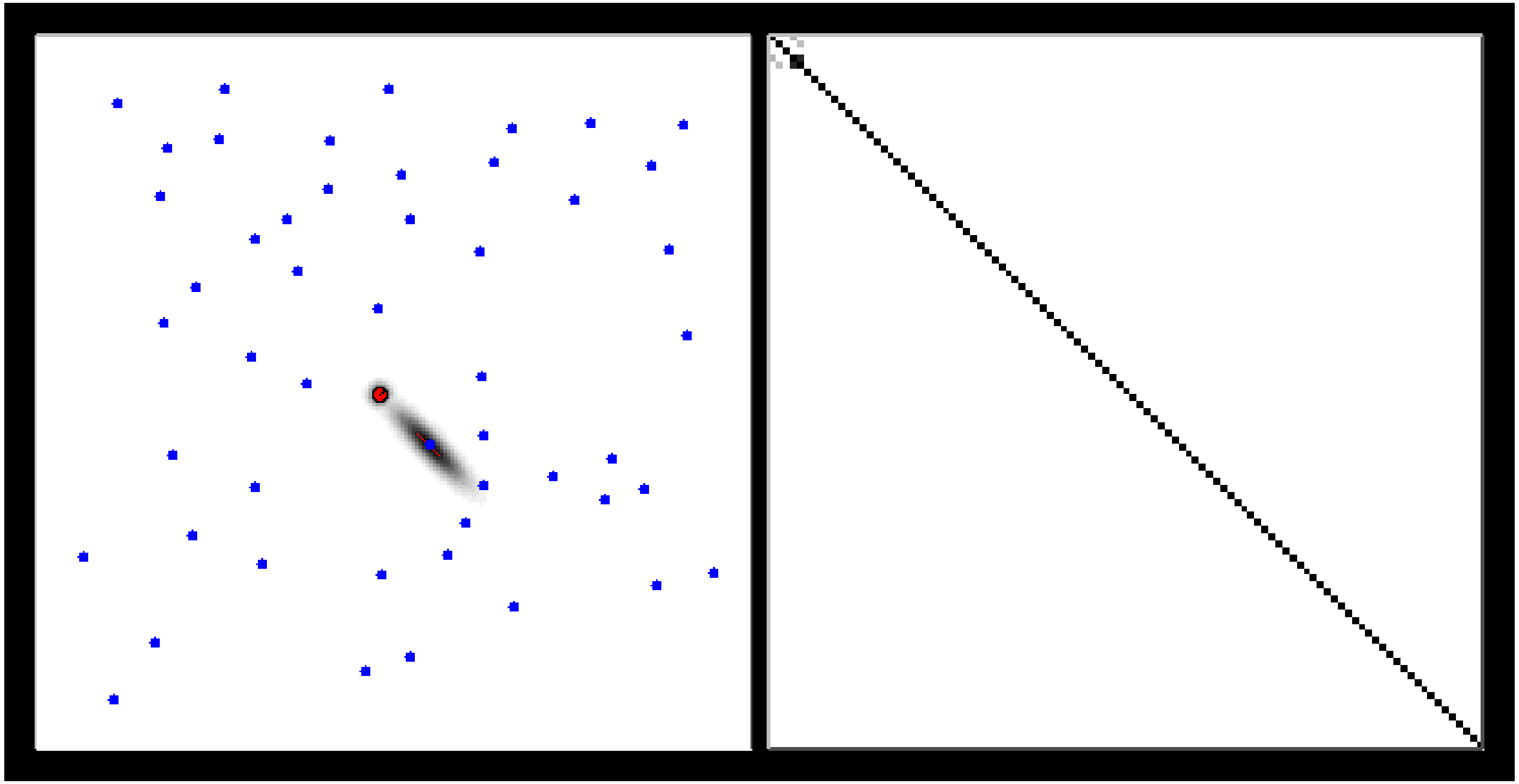
Classical Solution – The EKF



Blue path = true path   **Red path** = estimated path   **Black path** = odometry

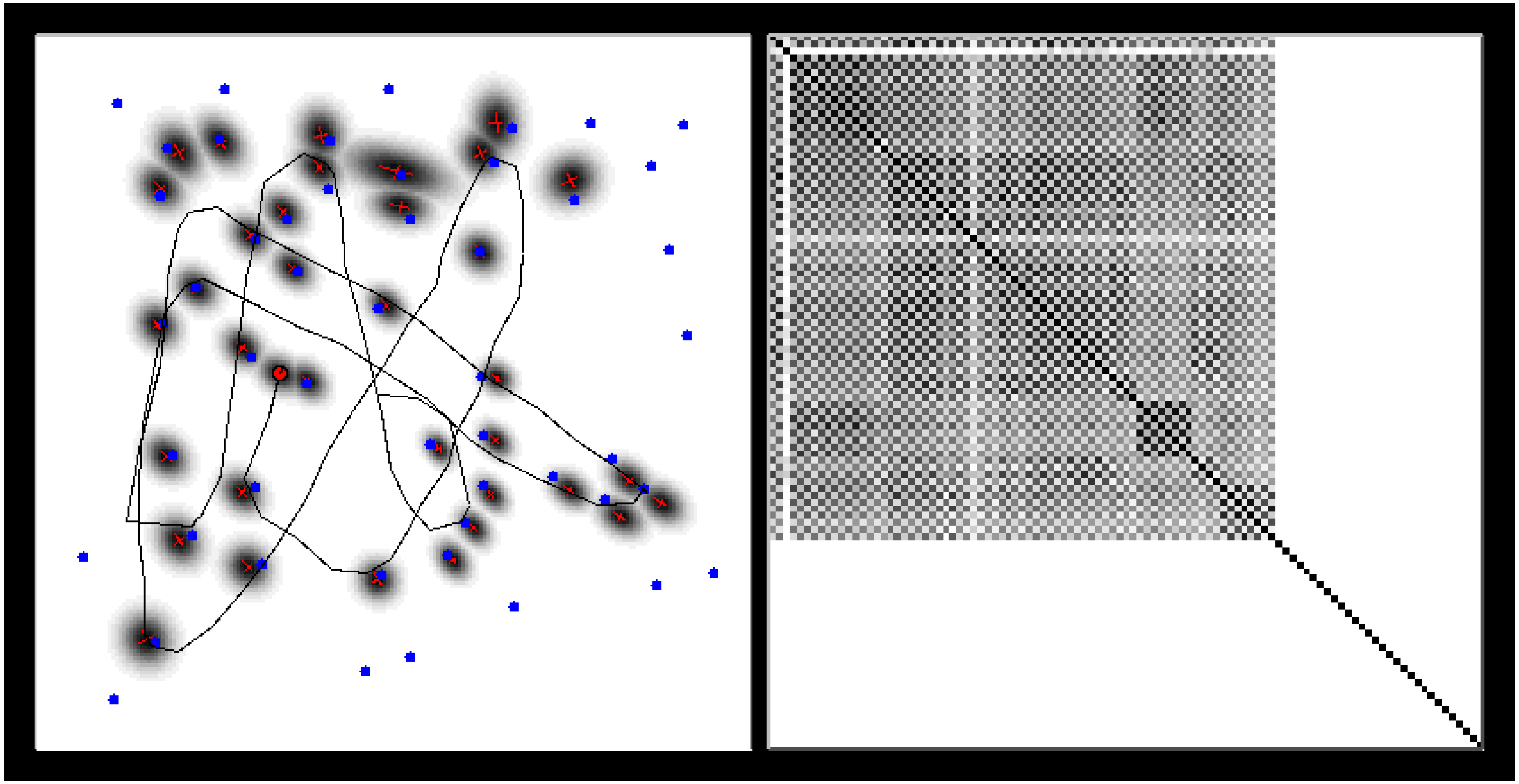
- Approximate the SLAM posterior with a high-dimensional Gaussian [Smith & Cheesman, 1986] …
- Single hypothesis data association

EKF-SLAM



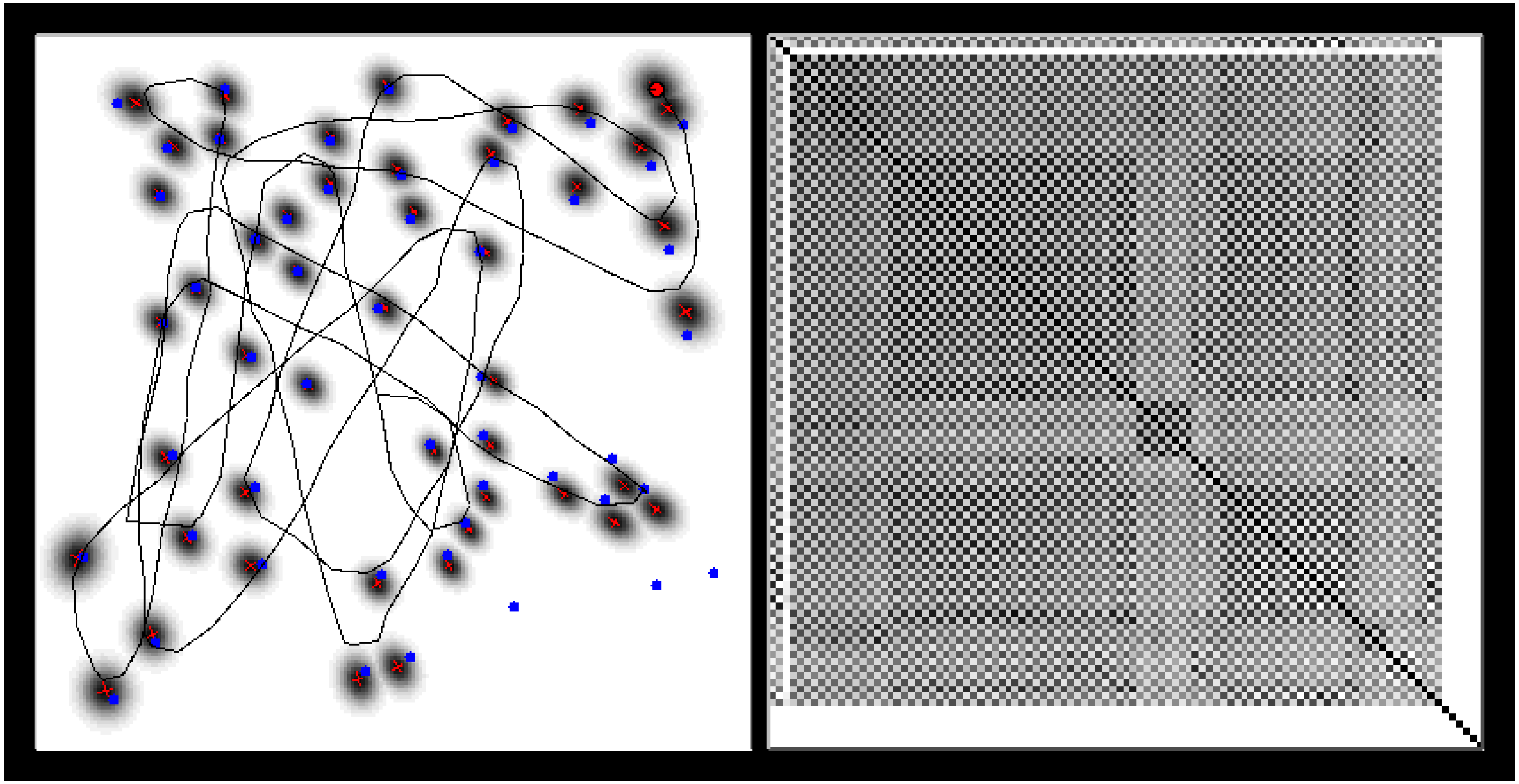
Map Correlation matrix

EKF-SLAM



Map Correlation matrix

EKF-SLAM



Map Correlation matrix

Properties of KF-SLAM (Linear Case) [Dissanayake et al., 2001]

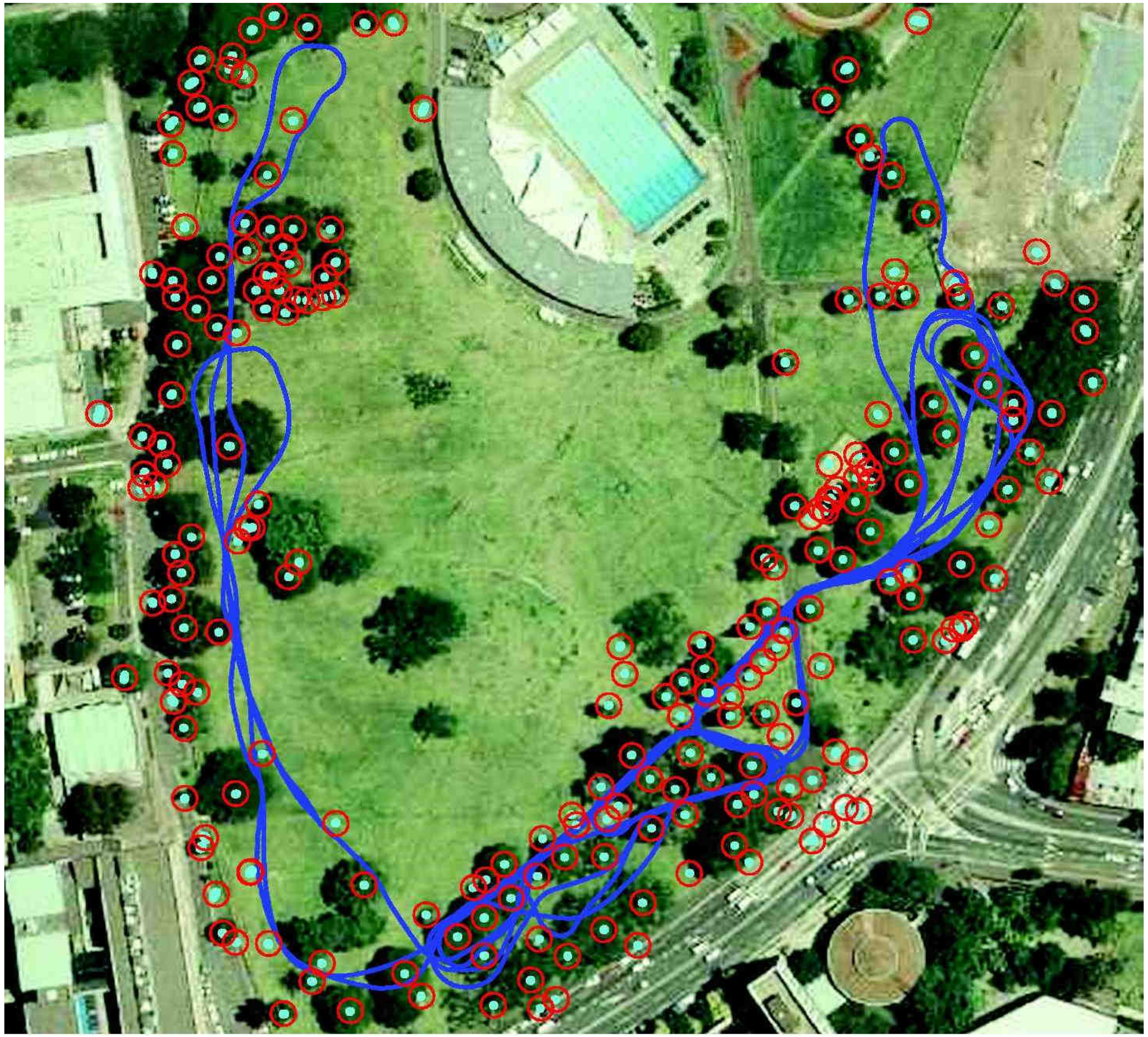
Theorem:

The determinant of any sub-matrix of the map covariance matrix decreases monotonically as successive observations are made.

Theorem:

In the limit the landmark estimates become fully correlated

Victoria Park Data Set



[courtesy by E. Nebot]

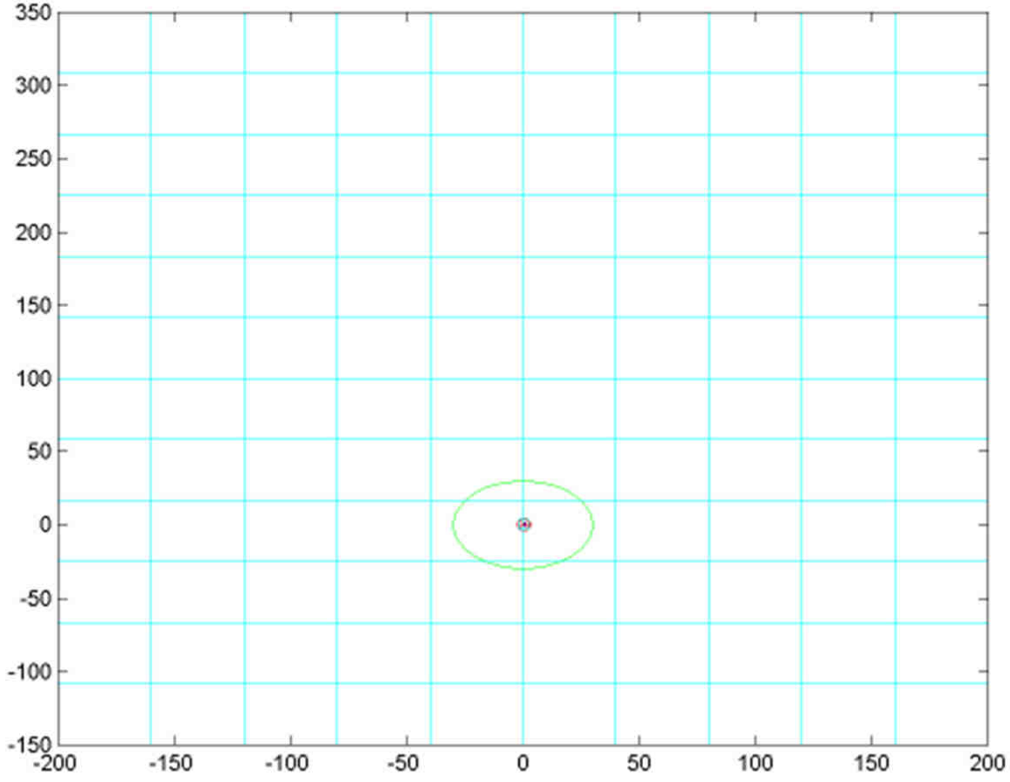
Victoria Park Data Set Vehicle

[courtesy by E. Nebot]

Data Acquisition

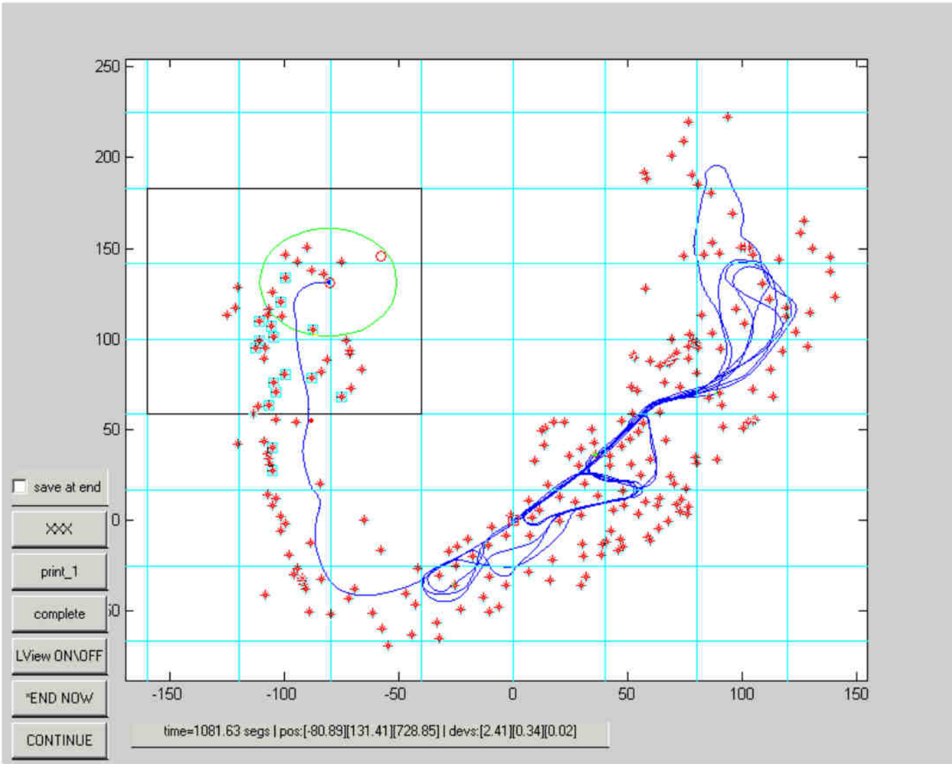
[courtesy by E. Nebot]

SLAM



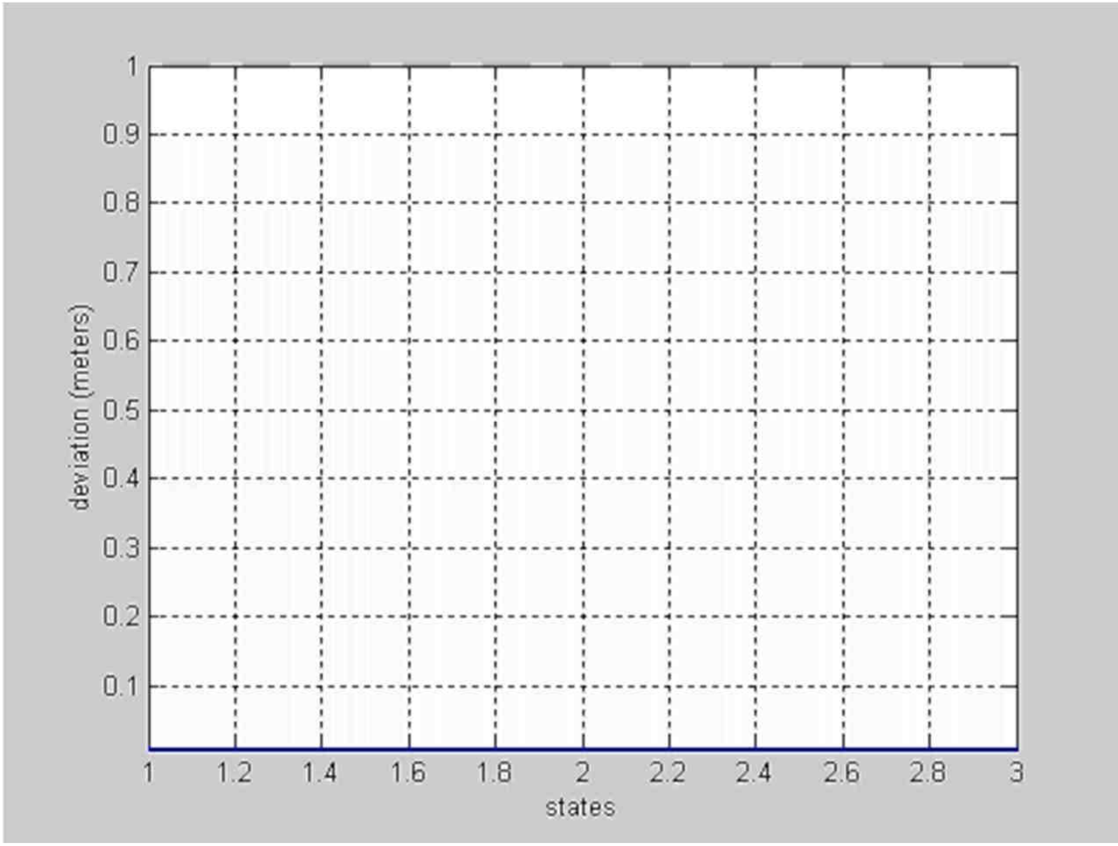
[courtesy by E. Nebot]

Map and Trajectory



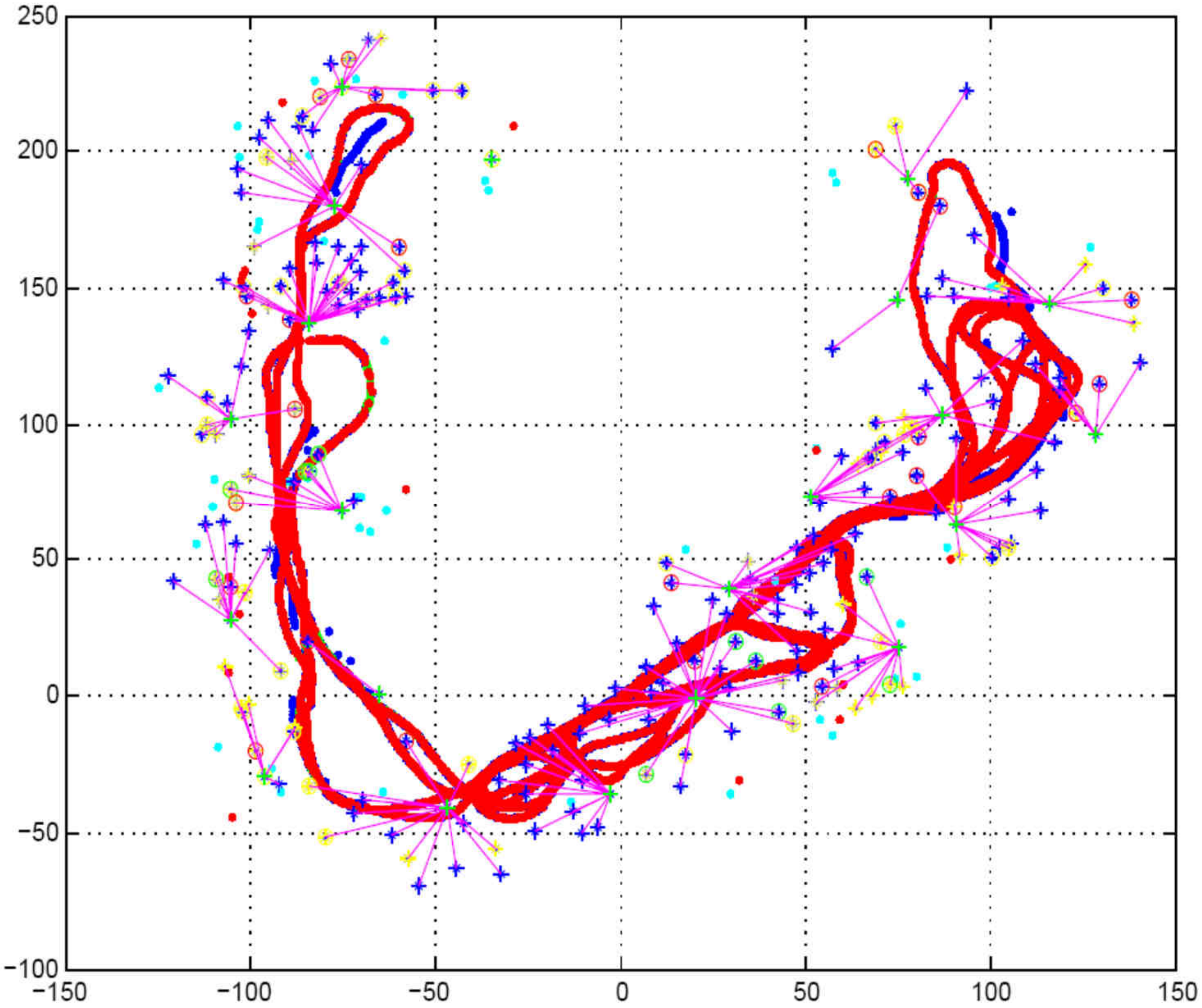
[courtesy by E. Nebot]

Landmark Covariance



[courtesy by E. Nebot]

Estimated Trajectory

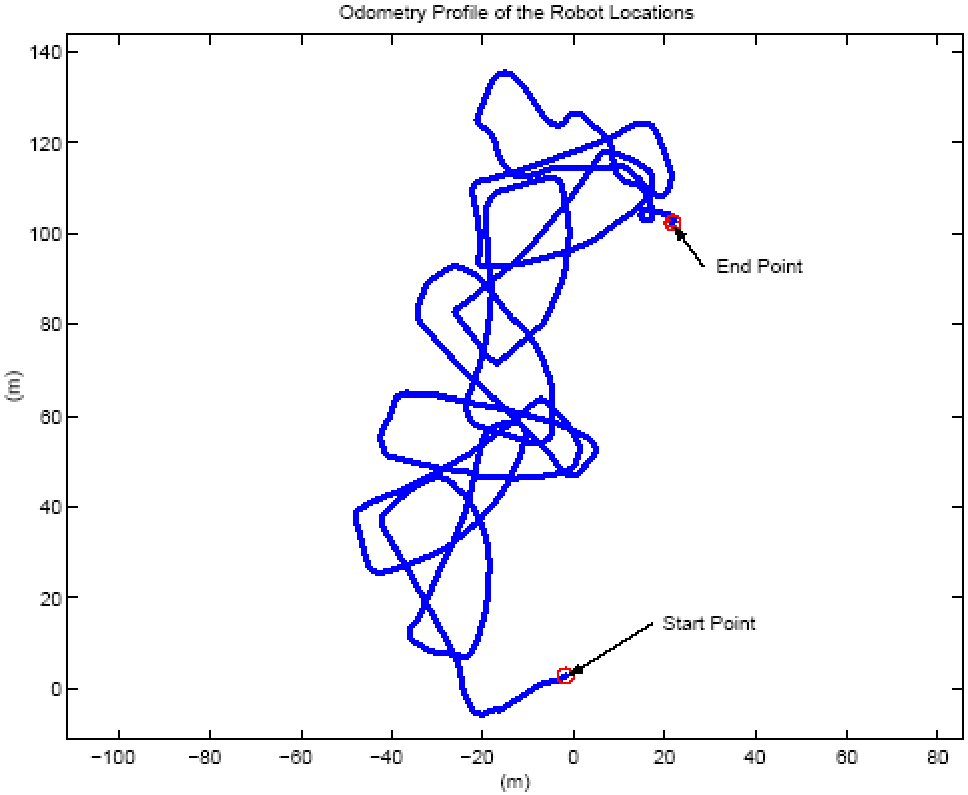


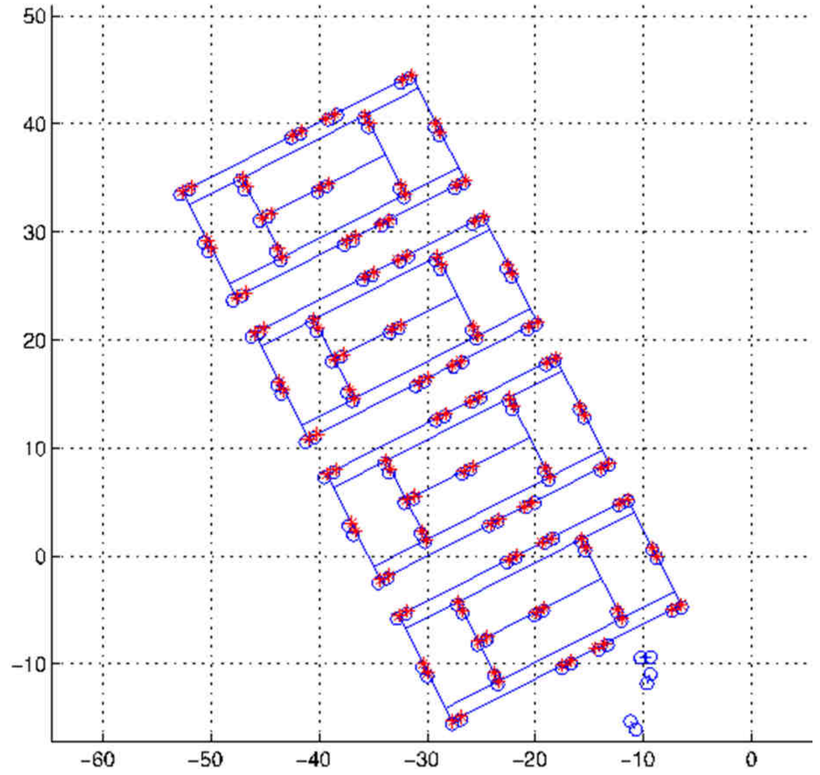
[courtesy by E. Nebot]

EKF SLAM Application

[courtesy by John Leonard]

EKF SLAM Application





odometry

estimated trajectory

[courtesy by John Leonard]

Approximations for SLAM

- Local submaps
  [Leonard et al.99, Bosse et al. 02, Newman et al. 03]
- Sparse links (correlations)
  [Lu & Milios 97, Guivant & Nebot 01]
- Sparse extended information filters
  [Frese et al. 01, Thrun et al. 02]
- Thin junction tree filters
  [Paskin 03]
- Rao-Blackwellisation (FastSLAM)
  [Murphy 99, Montemerlo et al. 02, Eliazar et al. 03, Haehnel et al. 03]

EKF-SLAM Summary

- Quadratic in the number of landmarks: $O(n^2)$
- Convergence results for the linear case.
- Can diverge if nonlinearities are large!
- Have been applied successfully in large-scale environments.
- Approximations reduce the computational complexity.

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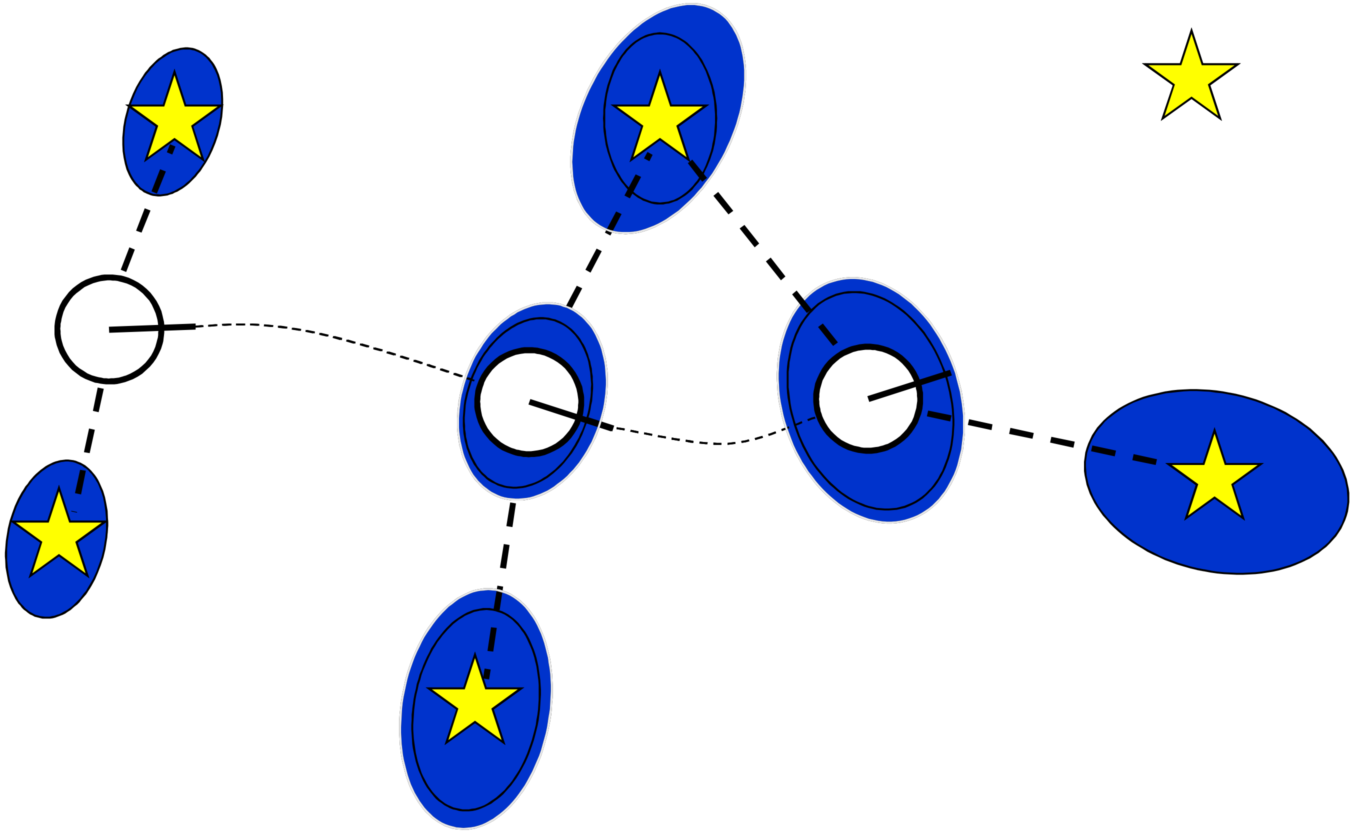

Robot path error correlates errors in the map

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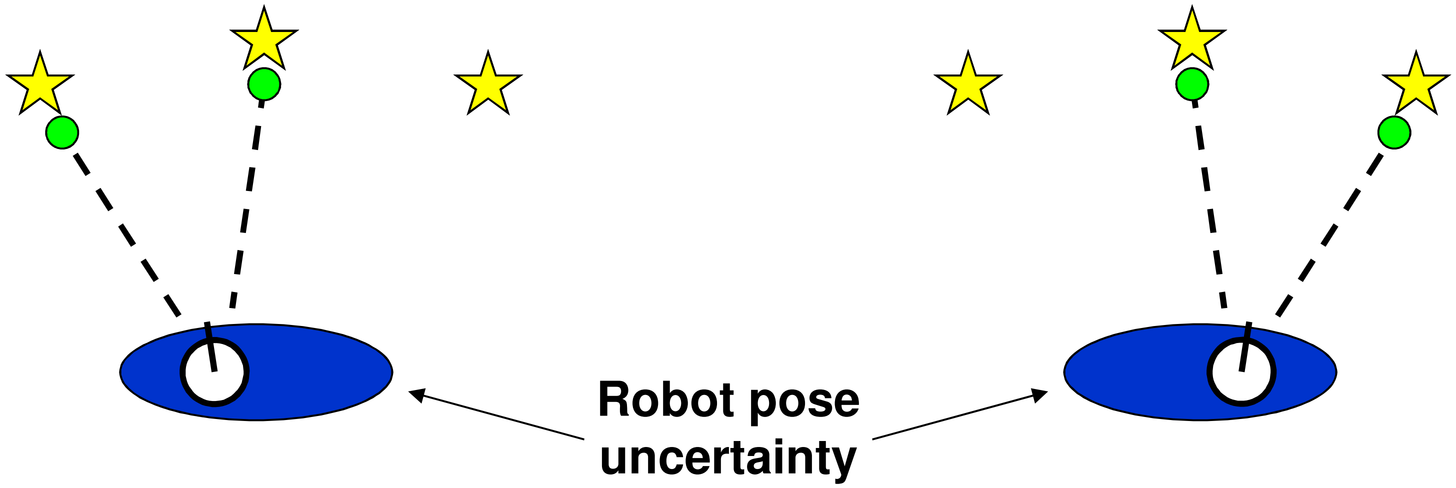
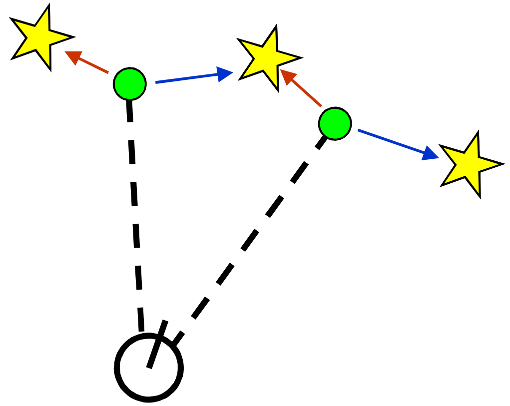


- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
- Pose error correlates data associations

Data Association Problem



- A data association is an assignment of observations to landmarks
- In general there are more than $\binom{n}{m}$ (n observations, m landmarks) possible associations
- Also called "assignment problem"

Particle Filters

- Represent belief by random samples
- Estimation of non-Gaussian, nonlinear processes

- Sampling Importance Resampling (SIR) principle
  - Draw the new generation of particles
  - Assign an importance weight to each particle
  - Resampling

- Typical application scenarios are tracking, localization, …

Localization vs. SLAM

- A particle filter can be used to solve both problems

- Localization: state space $<x, y, \theta>$

- SLAM: state space $<x, y, \theta, map>$
  - for landmark maps = $<l_1, l_2, \ldots, l_m>$
  - for grid maps = $<c_{11}, c_{12}, \ldots, c_{1n}, c_{21}, \ldots, c_{nm}>$

- **Problem:** The number of particles needed to represent a posterior grows exponentially with the dimension of the state space!

Dependencies

- Is there a dependency between the dimensions of the state space?
- If so, can we use the dependency to solve the problem more efficiently?

Dependencies

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- If so, can we use the dependency to solve the problem more efficiently?

- In the SLAM context
  - The map depends on the poses of the robot.
  - We know how to build a map given the position of the sensor is known.

Factored Posterior (Landmarks)

poses map observations & movements

$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$

$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(l_{1:m} \mid x_{1:t}, z_{1:t})$$

SLAM posterior

Robot path posterior

landmark positions

Does this help to solve the problem?

Factorization first introduced by Murphy in 1999

Factored Posterior (Landmarks)

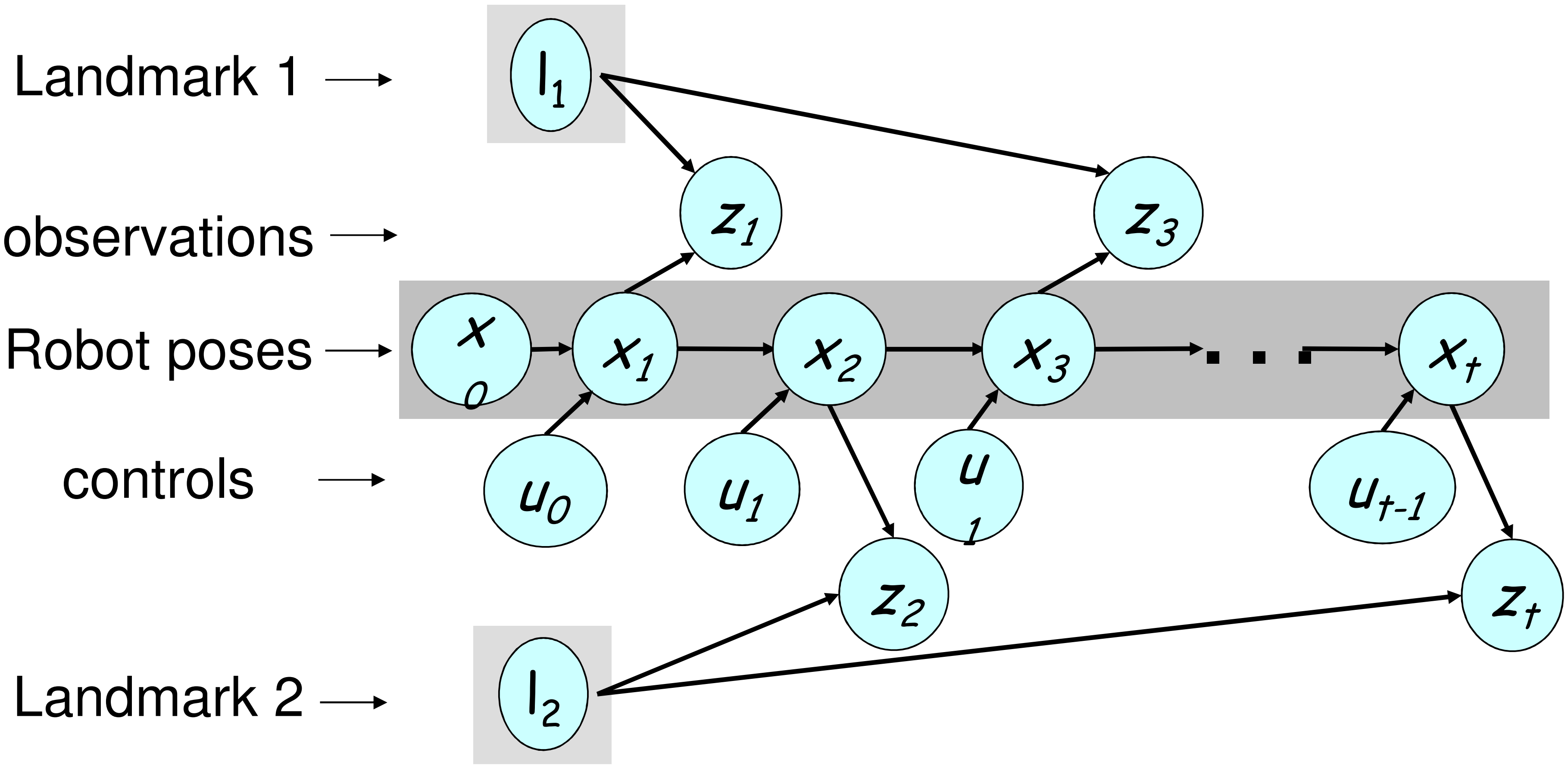
poses map observations & movements

$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$

$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(l_{1:m} \mid x_{1:t}, z_{1:t})$$

Factorization first introduced by Murphy in 1999

Mapping using Landmarks



Knowledge of the robot's true path renders landmark positions conditionally independent

Factored Posterior

$$
\begin{aligned}
& p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) \\
& \quad = \; p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(l_{1:m} \mid x_{1:t}, z_{1:t}) \\
& \quad = \; p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot \prod_{i=1}^{M} p(l_i \mid x_{1:t}, z_{1:t})
\end{aligned}
$$

Robot path posterior (localization problem)

Conditionally independent landmark positions

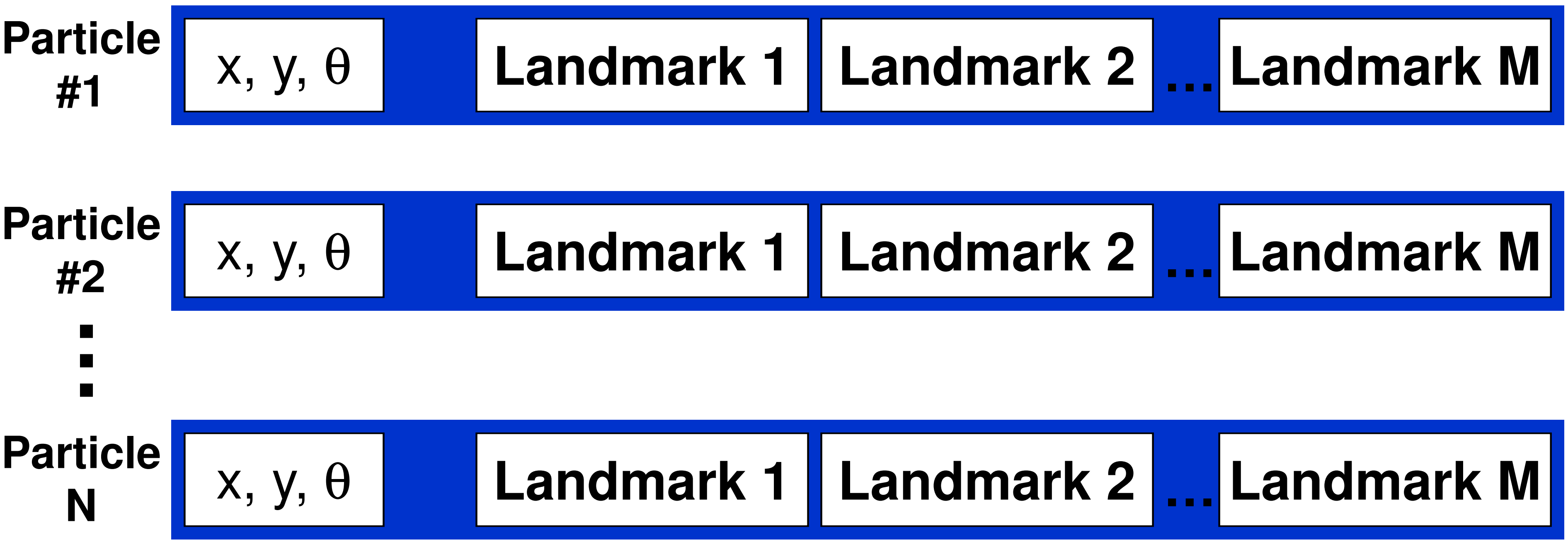
Rao-Blackwellization

$$p(x_{1:t}, l_{1:m} \mid z_{1:t}, u_{0:t-1}) =$$
$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot \prod_{i=1}^{M} p(l_i \mid x_{1:t}, z_{1:t})$$

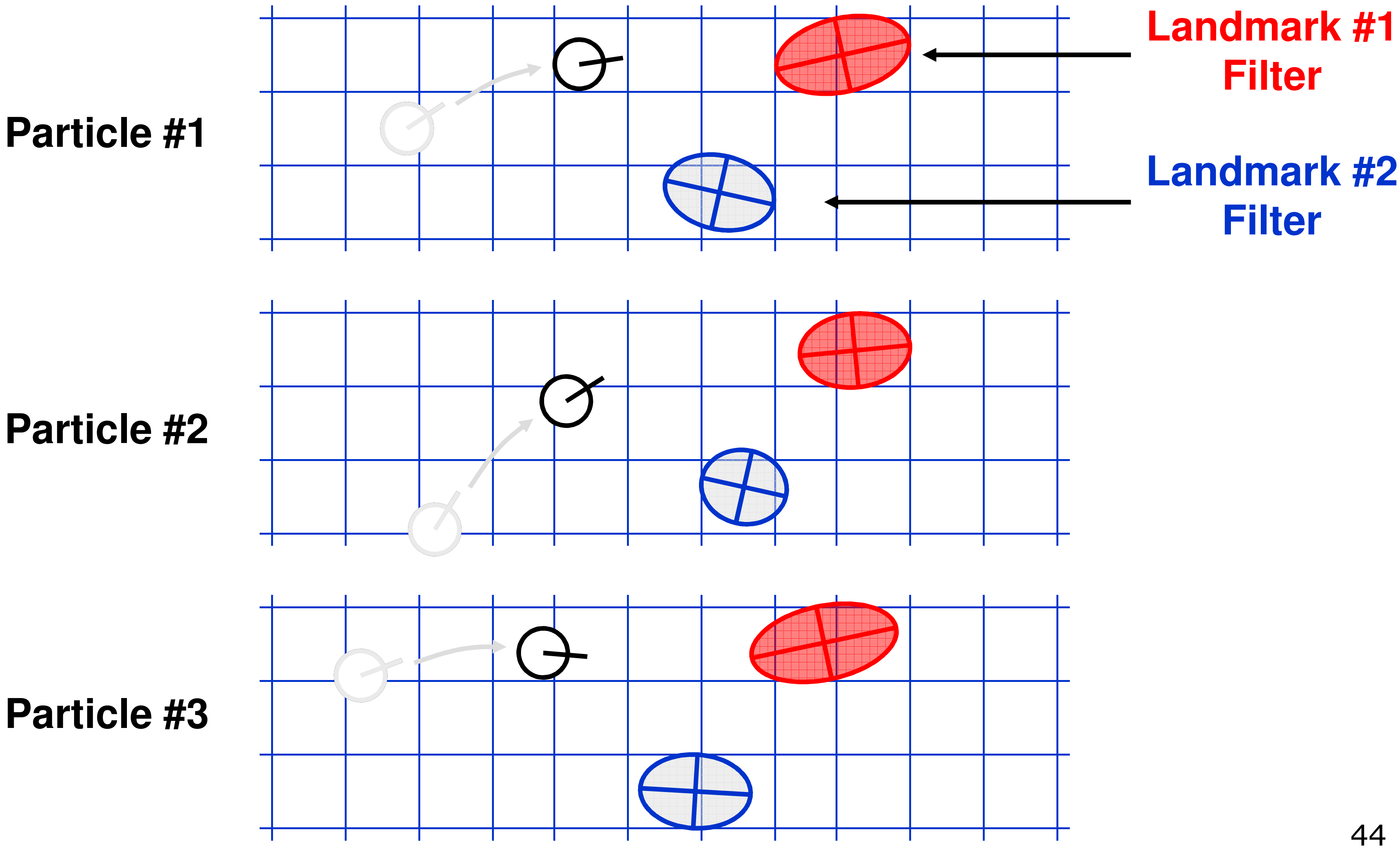
- This factorization is also called Rao-Blackwellization
- Given that the second term can be computed efficiently, particle filtering becomes possible!

FastSLAM

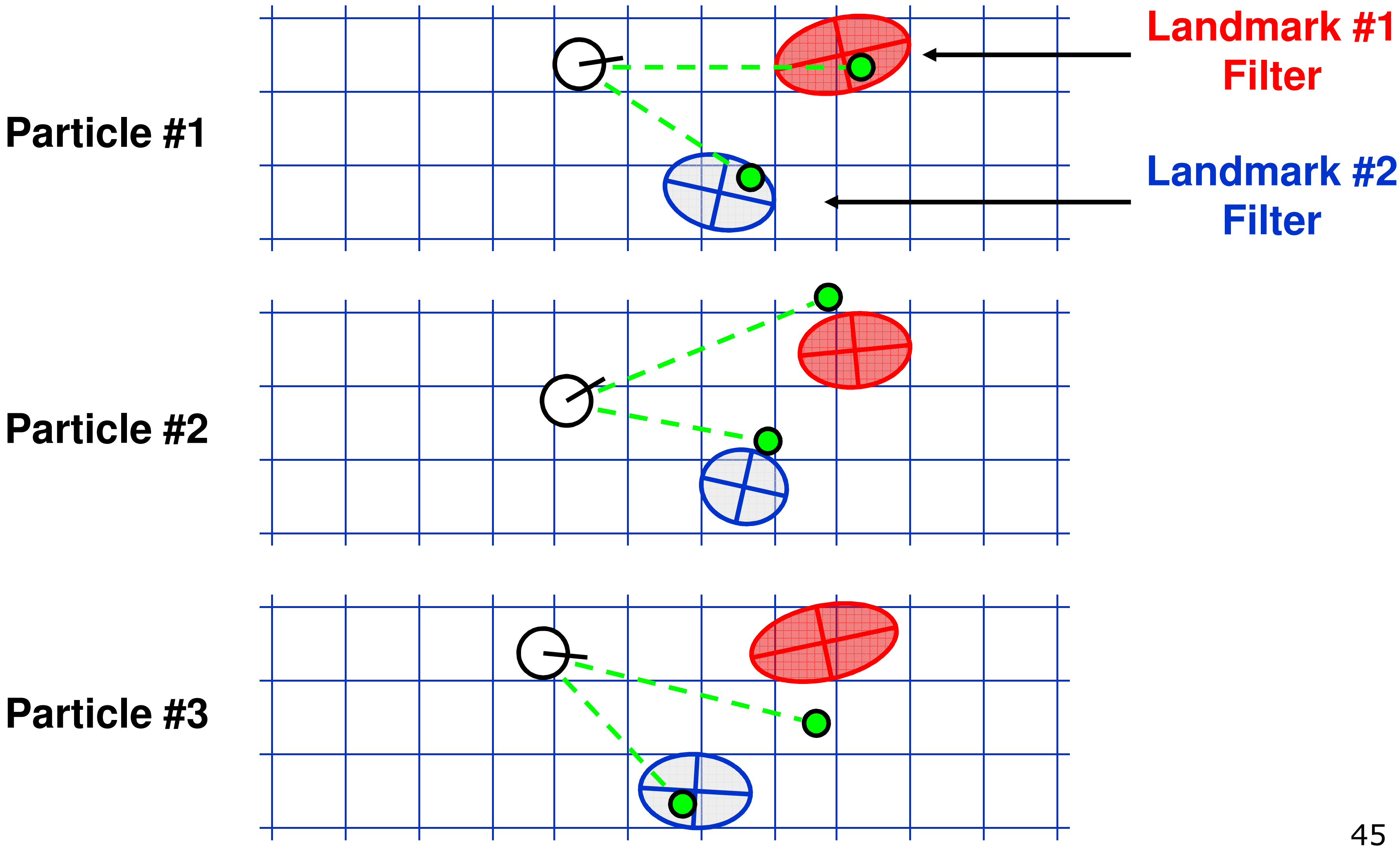
- Rao-Blackwellized particle filtering based on landmarks [Montemerlo et al., 2002]
- Each landmark is represented by a 2x2 Extended Kalman Filter (EKF)
- Each particle therefore has to maintain *M* EKFs



FastSLAM – Action Update



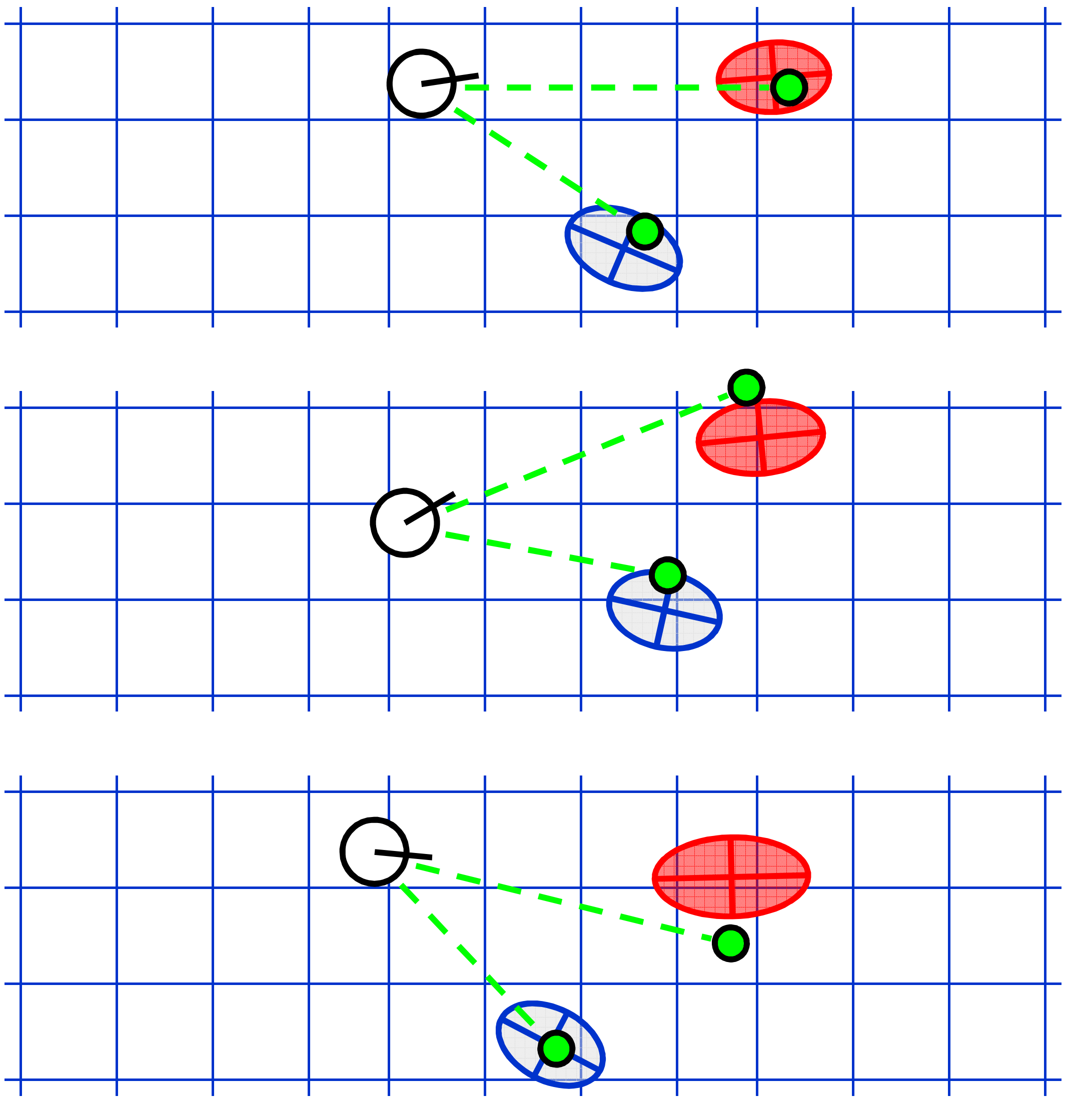
FastSLAM – Sensor Update



FastSLAM – Sensor Update

Particle #1

Weight = 0.8



Particle #2

Weight = 0.4

Particle #3

Weight = 0.1

FastSLAM Complexity

- Update robot particles based on control u_{t-1}

  $O(N)$
  Constant time per particle

- Incorporate observation z_t into Kalman filters

  $O(N \cdot \log(M))$
  Log time per particle

- Resample particle set

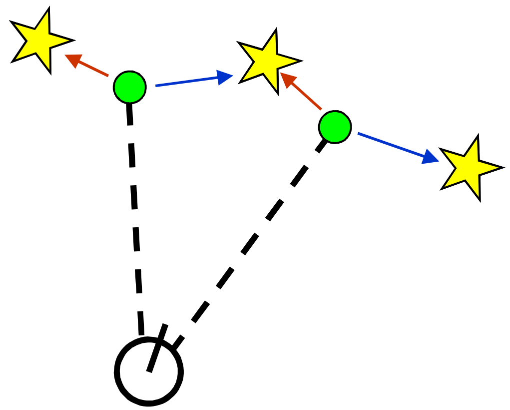
  $O(N \cdot \log(M))$
  Log time per particle

N = Number of particles
M = Number of map features

$O(N \cdot \log(M))$
Log time per particle

Data Association Problem

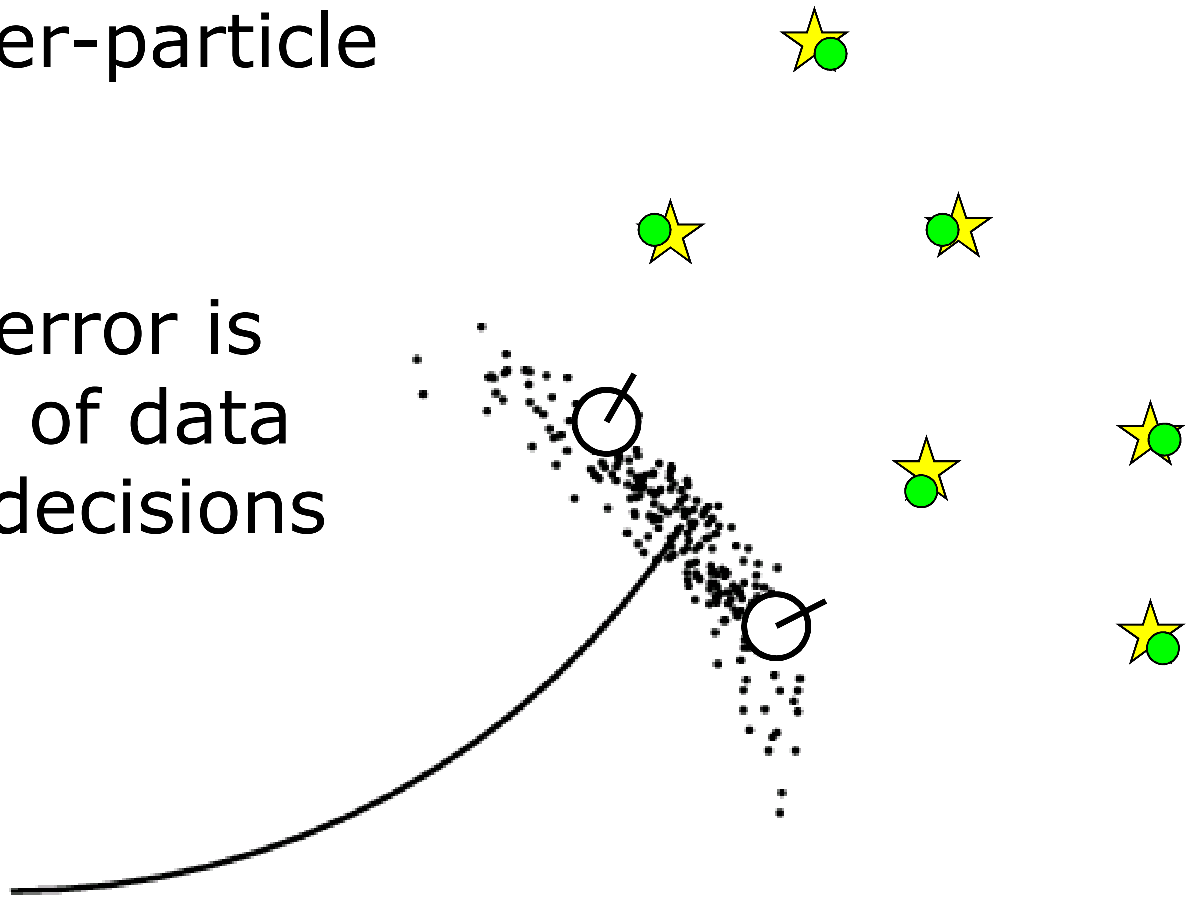
- Which observation belongs to which landmark?



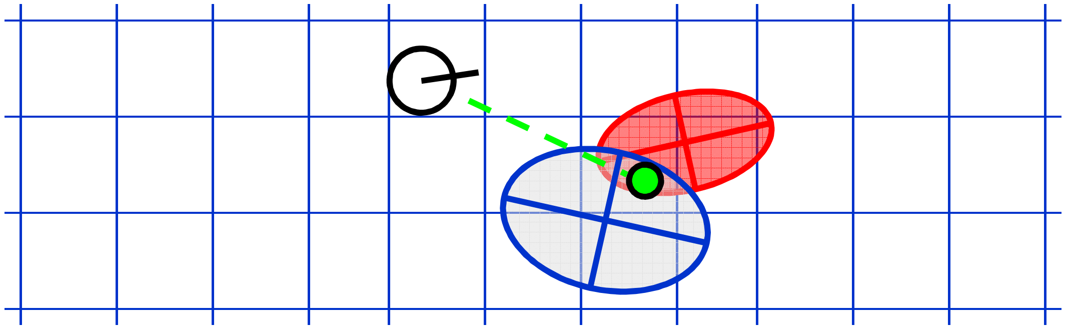
- A robust SLAM must consider possible data associations
- Potential data associations depend also on the pose of the robot

Multi-Hypothesis Data Association

- Data association is done on a per-particle basis
- Robot pose error is factored out of data association decisions



Per-Particle Data Association



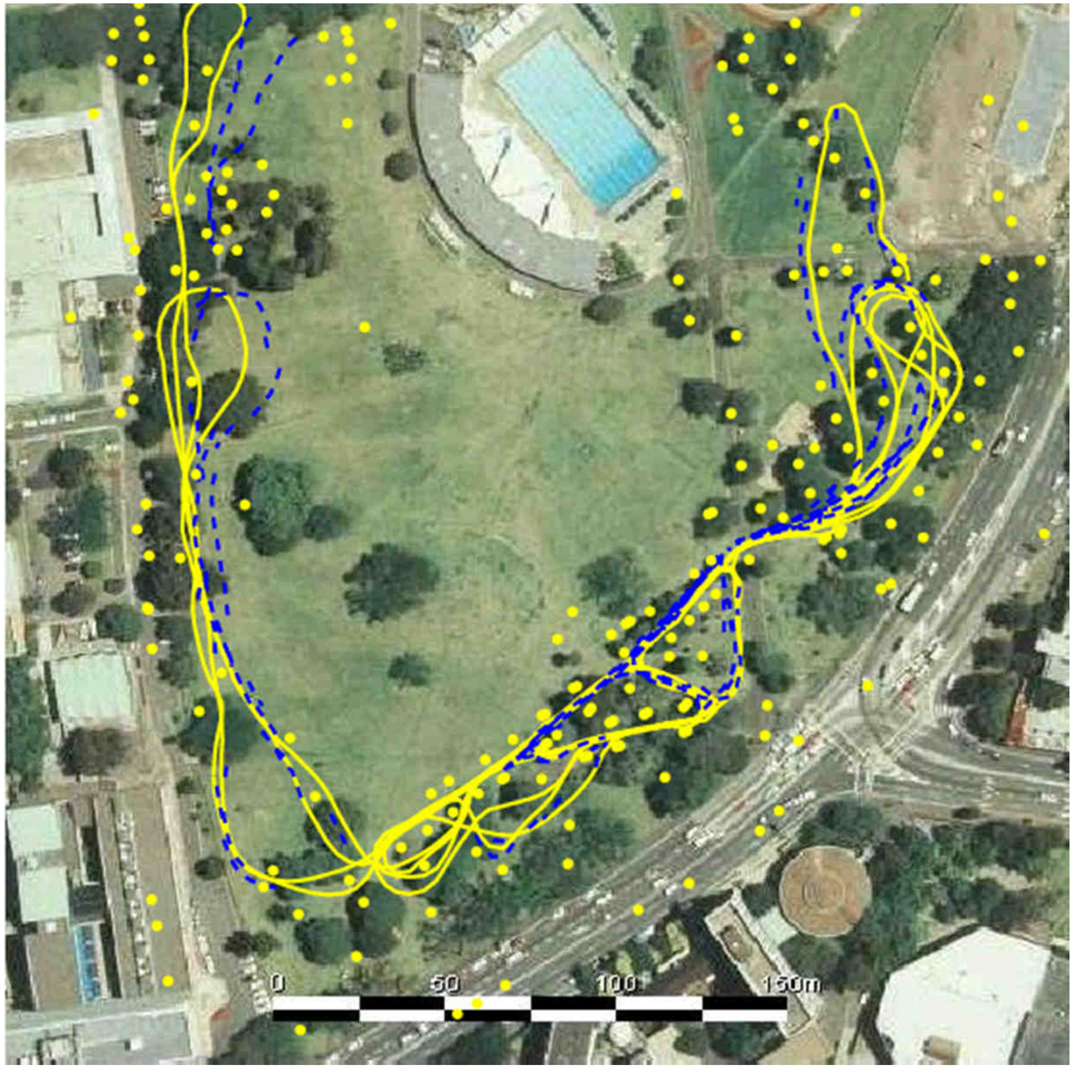
Was the observation generated by the red or the blue landmark?

P(observation|red) = 0.3 P(observation|blue) = 0.7

- Two options for per-particle data association
  - Pick the most probable match
  - Pick an random association weighted by the observation likelihoods
- If the probability is too low, generate a new landmark

Results – Victoria Park

- 4 km traverse
- < 5 m RMS position error
- 100 particles



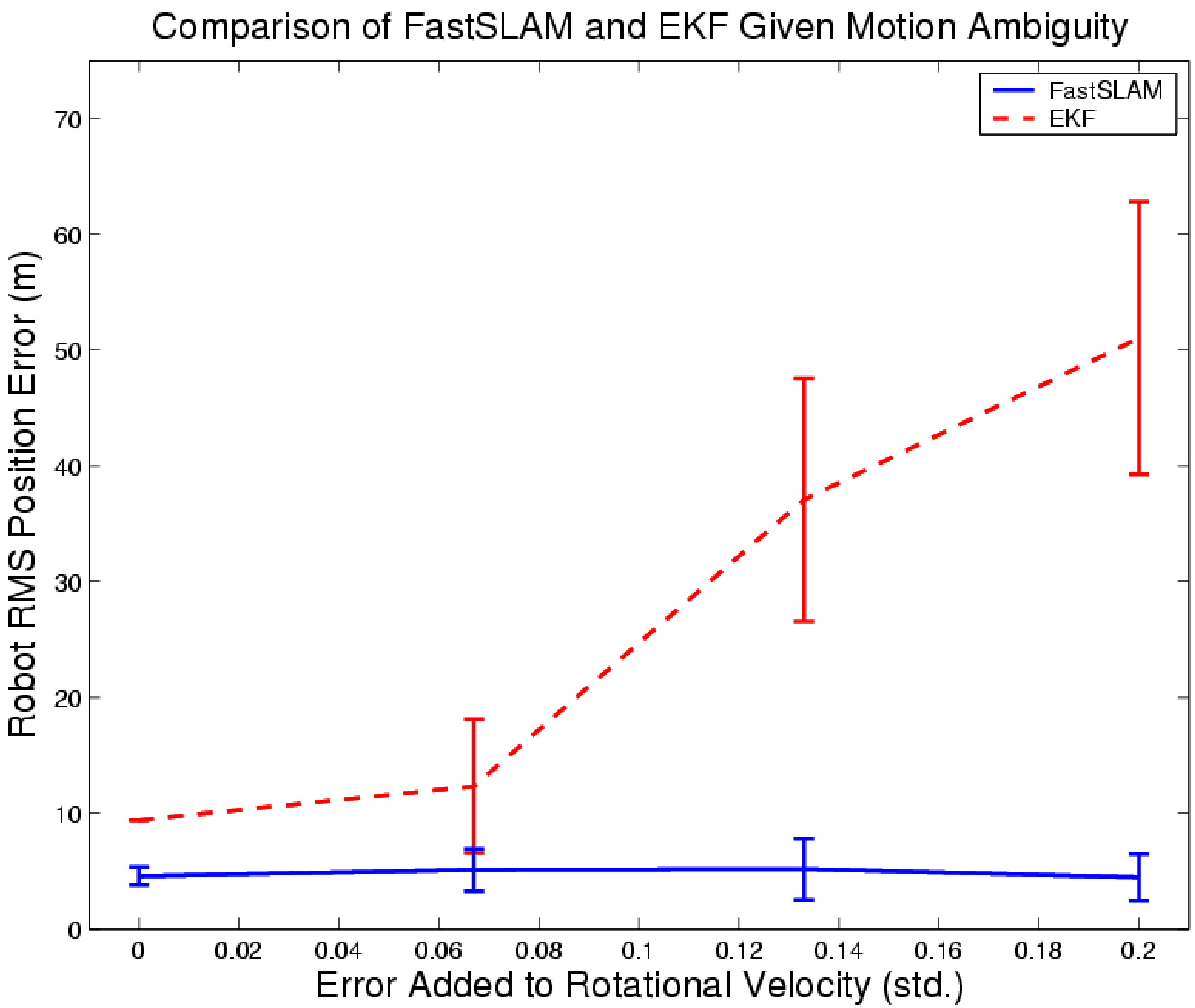
Blue = GPS
Yellow = FastSLAM

Dataset courtesy of University of Sydney

Results – Victoria Park

Dataset courtesy of University of Sydney

Results – Data Association



Results – Accuracy