CS 547: Sensing and Planning in Robotics

Analysis of 'Mobile Robot Localization by Tracking Geometric Beacons' by Leonard and Durrant-Whyte

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Summary

- Kalman filter-based localization i.e. the Belief prior and posterior are assumed to be Gaussian
- The world 'map' is very sparse
- Action model is rather simple
- Perception model is fine-tuned for sonar

Note: This is still Markov Localization

Terminology

- Robot state is $[x(k), y(k), \theta(k)]$, k is time
- The world map is a set of geometric beacons (or features) p_i, whose location is known in advance
- At each time step the robot observes these beacons. These observations are $z_j(k)$.

The Algorithm

- Initialize state $s = [x(k), y(k), \theta(k)]$ and associated covariance P
- Use the action (plant) model to predict state at $k+1$
  $i.e.$ $\hat{s} = [\hat{x}(k+1), \hat{y}(k+1), \hat{\theta}(k+1)]$, and associated covariance P
- Use the observation model to predict the measurements $\hat{z}_j(k+1)$
- Use sensors to record actual observations $z_j(k+1)$
- Compute diff between predicted and actual measurements (innovation)
- Outlier observations discarded using 'gating'
- Compute the Kalman gain as a function of the innovation
- Use KF equations to update the robot state and associated covariance

The Kalman filter as a Bayes Filter

- Kalman filters (KF) represent the posterior belief by a Gaussian (normal) distribution
- A 1D Gaussian is given by

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

- A Gaussian in d dimensions is given by

$$P(x) = \frac{1}{\sqrt{(2\pi)^d |\Sigma|}} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)}$$

- A KF is thus a particular parametric representation of a belief posterior

The Kalman filter is a Bayes Filter

... with the following assumptions:

- Initial belief $Bel(s^{(0)})$ is normal
- State at $t+1$ is a linear function of state at t

$$s^{(t+1)} = As^{(t)} + Ba^{(t)} + \epsilon^{(t)}_{action}$$

- Observations are linear in the state

$$o^{(t)} = Cs^{(t)} + \epsilon^{(t)}_{observation}$$

- $\epsilon^{(t)}_{action}$ and $\epsilon^{(t)}_{action}$ are both zero-mean random variables which are normally distributed with covariance Σ_{action} and $\Sigma_{observation}$

These assumptions guarantee that the posterior belief is a Gaussian, and there is a very efficient algorithm to compute the posterior called the Kalman Filter.

The Kalman filter

- $P(s|a, s')$ is a Gaussian whose mean is linear in a and s and whose covariance is Σ_{action}
- $P(o|s)$ is a Gaussian whose mean is linear in s and whose covariance is $\Sigma_{observation}$
- Thus each belief function uniquely characterized by its mean μ and covariance matrix Σ
- Computing the posterior amounts to updating the new μ and Σ from the old using either action or observation
- Key limitation: unimodality

The Kalman filter Algorithm

- Input: $Bel(s) = (\mu, \Sigma), d$
- Output: $Bel'(s) = (\mu', \Sigma')$

if $d = o$ then

$$K = \Sigma C^T (C \Sigma C^T + \Sigma_{observation})^{-1}$$
$$\mu' = \mu + K(o - C\mu)$$
$$\Sigma' = (I - KC)\Sigma$$

else if $d = a$ then

$$\mu' = A\mu + Ba$$
$$\Sigma' = A \Sigma A^T + \Sigma_{action}$$

return $Bel'(s) = (\mu', \Sigma')$

Mean and Variance

- Expectation of X (also called the mean μ)
  $E(x) = \int_{-\infty}^{\infty} xP(x)dx$
- Covariance of X and Y
  $Cov(x, y) = E(xy) - E(x)E(y)$
- Variance of X is its covariance with itself
  $Cov(x, x) = E(x^2) - E(x)^2$
- Variance of X is often denoted σ^2 where σ is called the standard deviation