

**CS 547: Sensing and Planning in Robotics**  
**Fall 2005**  
**Quiz 2: November 2, 2005**

**Notes:**

1. The quiz is open notes, not open book.
2. You may use a calculator.
3. You may not borrow notes or papers from others.
4. The quiz duration is 2.5 hours.
5. There are 10 pages in this enclosure.
6. Attempt all questions.
7. *Good Luck*

**Name:**\_\_\_\_\_

**SS#/USC ID#:**\_\_\_\_\_

<i>Problem</i>	<i>Score</i>	<i>Out Of</i>
1		15
2		15
3		15
4		20
Total		65

1. [15 points] Explain the following terms briefly (no more than 3-4 sentences each).

(a) Kalman filter

(b) Action model

(c) Probabilistic action model

(d) Sensor model

(e) Probabilistic sensor model

2. **[15 points]** Recall that a normal distribution is given by  $\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\frac{x-\mu^2}{\sigma^2}}$

We would like to apply Bayes rule to Gaussians. Suppose a mobile robot is confined to a long straight road. Its location (position along the road) is denoted  $x$ . Suppose that the initial estimate of the robot's location is  $x_{init} = 1000$ , but we happen to know that this estimate is uncertain. Based on this uncertainty we model our initial belief by a Gaussian with variance  $\sigma_{init}^2 = 900$ .

To find out more about the robot's location, we query a GPS receiver mounted on the robot. The GPS receiver measures a location of  $z_{GPS} = 1100$ . The GPS is known to have an error variance of  $\sigma_{init}^2 = 100$ .

- a. **[7 points]** Write the probability density functions of the prior  $p(x)$  and the measurement  $p(z|x)$ . Draw graphs showing each.

b. [8 points] Using Bayes rule compute the posterior  $p(z|x)$ . Is the posterior Gaussian? Justify your answer, and draw a graph of the posterior.

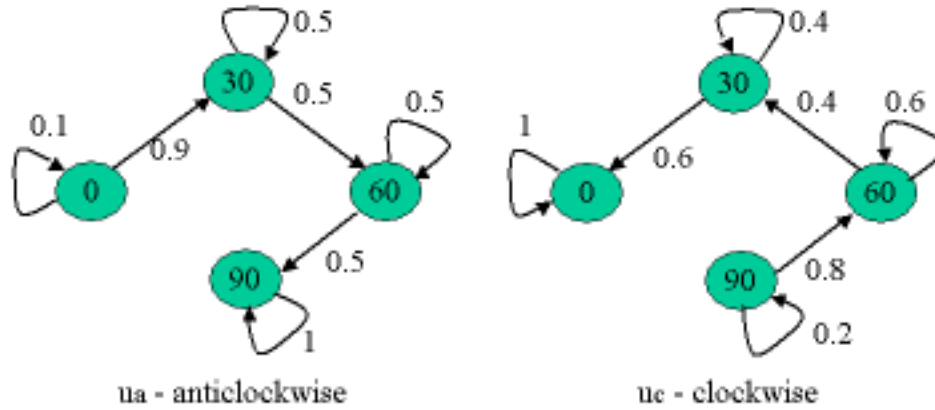
3. **[15 points]** Imagine a robot that can move back and forth along a narrow road (i.e. along a line) and can turn in place. Denote the robot's position along the road by  $x$  and its angular orientation by  $\theta$ . The robot is equipped with two controls  $v$  and  $\omega$ , for linear and angular velocity respectively. We wish to derive a velocity-based probabilistic action model for this robot following the treatment discussed in class.

a. **[3 points]** Clearly state the inputs and the output for such a model.

b. **[3 points]** Enumerate three assumptions one might make in such a model.

c. [**9 points**] Under the assumptions enumerated in the previous part, write a procedure which implements a velocity-based probabilistic action model for this robot. Like the treatment in class, you may assume that you are given a function **prob**( $a, b$ ) which computes the probability of its argument  $a$  under a zero-centered distribution with standard deviation  $b$ .

4. [20 points] A monitoring camera is installed such that it can be rotated clockwise ( $u_c$ ) or anticlockwise ( $u_a$ ). The actuator responsible for camera rotation is unreliable. The camera can exist at one of four angular positions  $\theta = 0^\circ, 30^\circ, 60^\circ, 90^\circ$ . The state transition diagram showing the effect of action  $u_a$  and  $u_c$  on the angular position of the camera is shown in the figure below.



- a. [5 points] Draw an unbiased prior over the variable  $\theta$ .

b. [6 points] Starting with an unbiased prior the camera is subjected to the following action sequence  $(u_c, u_c, u_c)$ . Compute and graph each succeeding posterior over  $\theta$  on the graphs below.

c. **[6 points]** Starting with an unbiased prior the camera is subjected to the following action sequence  $(u_a, u_c, u_a)$ . Compute and graph each succeeding posterior over  $\theta$  on the graphs below.

d. **[3 points]** Starting with an unbiased prior, which of these two action sequences leads to a better estimate of the actual camera angle ? Briefly justify your answer.