Smoother based 3D Attitude Estimation for Mobile Robot Localization*

Stergios I. Roumeliotis[1], Gaurav S. Sukhatme[2] and George A. Bekey[1,2]

stergios|gaurav|bekey@robotics.usc.edu

[1]Department of Electrical Engineering
[2]Department of Computer Science
Institute for Robotics and Intelligent Systems
University of Southern California
Los Angeles, CA 90089-0781

Abstract

The mobile robot localization problem is decomposed into two stages; attitude estimation followed by position estimation. The innovation of our method is the use of a smoother, in the attitude estimation loop that outperforms other Kalman filter based techniques in estimate accuracy. The smoother exploits the special nature of the data fused; high frequency inertial sensor (gyroscope) data and low frequency absolute orientation data (from a compass or sun sensor). Two Kalman filters form the smoother. During each time interval one of them propagates the attitude estimate forward in time until it is updated by an absolute orientation sensor. At this time, the second filter propagates the recently renewed estimate back in time. The smoother optimally exploits the limited observability of the system by combining the outcome of the two filters. The system model uses gyro modeling which relies on integrating the kinematic equations to propagate the attitude estimates and obviates the need for complex dynamic modeling. The Indirect (error state) form of the Kalman filter is developed for both parts of the smoother. The proposed approach is independent of the robot structure and the morphology of the ground. It can easily be transfered to another robot which has an equivalent set of sensors. Quaternions are used for the 3D attitude representation, mainly for practical reasons discussed in the paper. The proposed innovative algorithm is tested in simulation and the overall improvement in position estimation is demonstrated.

1 Introduction

In this paper we consider the problem of localizing a mobile robot on uneven terrain. Specifically, we are motivated by the problem of localizing the next generation of robot rovers [11] on the surface of Mars. The unique constraints of the problem are, 1. No Global Positioning System (GPS) is available, 2. Absolute orientation of the robot can only be sensed intermittently and 3. While in motion, rates of roll, pitch and yaw available from gyroscopes are subject to drift and noise. We present here, a localization algorithm subject to these constraints that generalizes across different mobile robot platforms with varying kinematics and dynamics. Future missions to Mars will demand long traverses (several km) of rovers to sites of scientific interest. In order to autonomously perform their scientific tasks, these rovers need to localize themselves. It should be noted, however, that localization is a problem that concerns every autonomous vehicle. The two basic techniques that have been developed to tackle this problem are: 1. **Relative (local) localization** which consists of evaluating the position and orientation through integration of encoder and inertial sensor data. The integration is started from the initial position and orientation and is continuously updated in time. This technique is often called dead-reckoning and relies either on odometry (wheel encoders) or inertial navigation systems (gyroscopes and accelerometers). Though the technique is simple, it is prone to error due to imprecision in modeling, noise, drift and slip. Substantial improvement is provided by applying Kalman filtering techniques [10]; 2. **Absolute (global) localization** which permits the robot to determine its position directly using navigation beacons, active or passive landmarks, map matching or satellite-based signals like GPS. Global sensor measurements can drastically increase the accuracy of the estimate and keep the associated uncertainty within certain bounds.

In this paper we address the problem of 3D localization for mobile robots in the absence of absolute positioning information. We concentrate on bounding the attitude uncertainty through periodic use of absolute attitude measurements. As a consequence the position estimate degrades slowly compared to the case when no ab-

*This work is supported in part by JPL, Caltech under contract #959816 and DARPA under contracts #F04701-97-C-0021 and #DAAE07-98-C-L026

solute orientation information is available. The attitude estimate relies on the gyros when the vehicle is in motion while a tri-axial accelerometer is used as an absolute orientation measuring device (roll and pitch) in conjunction with a sun sensor (yaw) when the vehicle is at stop. At the end of each interval of motion a smoother is used which propagates the new absolute orientation information backwards using the previously acquired gyro information. This lowers the uncertainty of the attitude estimate throughout the interval of smoothing; that is when the vehicle was in motion. Both the forward and backward estimators are Indirect (error state) Kalman filters and gyro modeling is used instead of a dynamic model of the robot. Smoothing (which is being applied to mobile robot localization for the first time), has been successfully used in the post-flight construction of the attitude profile of a spacecraft in order to support the integration of data from a space-born sensor like a camera for example.

In the next section we survey previous work in robot localization. Section 3 examines the dependence of the position estimate on the attitude estimate. We discuss the various attitude measuring devices used, the rationale behind dynamic model replacement and the Indirect Kalman filter and a basic gyro model. Section 4 contains a derivation of the error state equations for the 3-D case using unit quaternions. The linear time-variant equations of the system model and the non-linear equations of the observation model are derived. An Indirect Kalman filter based on these models is developed. The improvement due to the smoother is demonstrated. Section 5 shows how the position is updated using the improved attitude estimates and section 6 summarizes the contributions of this work and discusses future avenues of research.

2 Previous Work

In order to deal with systematic errors in indoor applications, a calibration technique called the UMBmark test is given in [3]. [4] discusses a technique called gyrodometry, which uses odometry data most of the time, while substituting gyro data only during brief instances (e.g. when the vehicle goes over a bump) during which gyro and odometry data differ drastically. This way the system is kept largely free of the drift associated with the gyroscope. A complementary Kalman filter [6] is used in [9] to estimate the robot's attitude from the accelerometer signal during low frequency motion and the gyro signal during high frequency motion. The attitude information is then used to calculate a position increment. In [1] the authors use a low cost INS system (3 gyroscopes, and a triaxial accelerometer) and 2 tilt sensors. Their approach is to incorporate in the system *a priori* information about the error characteristics of the inertial sensors and to use this directly in an Extended Kalman Filter (EKF) to estimate position.

Examples of absolute localization include [14] in which the localization algorithm is formalized as a tracking problem, employing an EKF to match beacon observations to a map in order to maintain an estimate of the position of the mobile robot; [2] in which the authors use an EKF to fuse odometry and angular measurements of known landmarks and [23] in which a Bayesian approach is used to learn useful landmarks for localization.

Most of the above approaches limit themselves to the case of planar motion. In addition, their accuracy depends heavily on the presence of some form of an absolute positioning system. We consider motion on uneven terrain (3D localization) and propose an estimation algorithm that is capable of incorporating absolute position measurements but is also able to provide reliable estimates in the absence of externally provided positioning information. Our method performs attitude estimation using an Indirect Kalman filter that operates on the error state.

3 Localization and Attitude Estimation

In this section we examine the relation between the attitude estimate and the position estimate. We use an experimental Mars rover prototype (Rocky 7 [11]) as the motivating example throughout this paper. The assumption is that the robot has wheel encoders, 3 gyros, 3 accelerometers and a sun sensor. Since there is no device measuring the absolute position of the rover (there is no GPS on Mars), the position can only be estimated through the integration of the accelerometer signal which has bias and noise. Consider also, that the propagation of the position relies upon the attitude estimate. Small errors in orientation fast become large errors in position. Formally speaking, the position is not observable and thus the uncertainty of its estimate will grow unbounded. The most promising course of action with this set of sensors is to focus on gaining a very precise attitude estimate. As a result the position uncertainty will grow at a slower rate. The attitude estimate is used twice during position estimation:

1. The accelerometer measures both the vehicle's acceleration and the projection of the gravitational acceleration on the accelerometer local frame. The relation between these is described by:

$$\ddot{\vec{p}}(t) = \vec{f}(t)/m = \vec{a}_{acc}(t) - A(q(t))\vec{g} \qquad (1)$$

   where $\ddot{\vec{p}}$ is the vehicle's (non-gravitational) acceleration, $\vec{a}_{acc}$ is the measurement from the 3-axis accelerometer and $\vec{g}$ is the gravitational acceleration. Precise knowledge of the orientation matrix $A(q)$ is mandatory to extract $\ddot{\vec{p}}$ accurately.

2. The next step requires integration of $\vec{\ddot{p}}$ to derive the position. $\vec{\ddot{p}}$ is local (i.e. expressed in a coordinate frame attached to the robot) and in order to calculate the position in global coordinates the attitude information is once again required:

$$\vec{p}(t) = \int_0^t dt' \int_0^{t'} A(q(t''))\vec{\ddot{p}}(t'')dt'' \tag{2}$$

3.1 Attitude Measuring Devices

The on-board gyroscopes can be used to calculate the attitude of the vehicle by integrating their signal. On the other hand, the sun sensor directly measures the values of the two components of a two-dimensional vector. This vector is the projection of the unit vector towards the sun on the sun sensor plane. Another sensory input of the same nature is required in order to satisfy attitude observability requirements. While the accelerometer is mainly used to advance the position estimate (Equations 1,2) it can also be used in an alternative way. An accelerometer can measure the local gravitational acceleration, a three-dimensional vector parallel to the local vertical. This provides another orientation fix independent from the sun and thus makes the vehicle's attitude observable. When the vehicle is stopped the accelerometer measures only the gravitational acceleration namely $\vec{a}_{acc} = A(q)\vec{g}$. The roll and pitch of the vehicle can thus be precisely calculated. The sun sensor provides the yaw measurement and thus the matrix $A(q)$ is observable and precisely known when at stop.

This method fails when the rover is in motion. The gravity vector is then "contaminated" by the non-gravitational acceleration of the vehicle (Equation 1). The gravity vector could be extracted while the vehicle is moving if an independent measurement of its own acceleration was available. Research efforts [24, 9] have tried to address this problem using additional information from odometry. We believe that these approaches are sufficient for indoor applications and can deal with cases of motion over small objects but are not accurate enough for general outdoor environments mainly because of the limited accuracy of the estimates of the non-gravitational acceleration. A more thorough consideration of the problem would require dynamic modeling of the vehicle. An estimator that incorporates a dynamic model of the vehicle [21] could estimate its non-gravitational accelerations.

3.2 Dynamic Model Replacement

In our approach we avoid dynamic modeling and restrict ourselves to **use the accelerometer only when the rover is at stop**. The reasons for avoiding dynamic modeling are: 1. generality, 2. practical estimator size, 3. reported poor payoffs [12] due to dynamic modeling, and 4. complexity. Due to space constraints, we do not discuss these further, the interested reader is referred to [19, 20] for further details.

3.3 The Indirect-feedback Kalman Filter

As mentioned before, Kalman filtering has been widely used for localization purposes. The kinds that usually appear in mobile robot applications are the linear Kalman filter and the Extended Kalman filter (EKF) forms of the full state Kalman filter. In this work we choose to use the error-state form of both the linear Kalman filter and EKF. In the error-state (indirect) formulation, the *errors* in orientation are among the estimated variables, and each measurement presented to the filter is the difference between the INS and the external source data (i.e from absolute orientation sensors). In the following section we derive the equations needed for such a formulation. The primary reasons to pick this formulation are 1. No explicit modeling of the vehicle dynamics is needed, 2. The filter runs at a relatively low frequency, and 3. In case the filter fails, integrated estimates of the INS data continue to be available.

In the feedback form of the Indirect-feedback Kalman filter the updated error estimate is actually fed back to the INS to correct its "new" starting point, i.e. the state that the integration for the new time step will start from. The rationale behind the Indirect Kalman filter as well as the feedback form are discussed in further detail in [19, 20].

3.4 Gyro Modeling

A great difficulty in all attitude estimation approaches that use gyros, is the low frequency noise component, also referred to as bias or drift that violates the white noise assumption required for standard Kalman filtering. This problem has attracted the interest of many researchers since the early days of the space program [16]. Inclusion of the gyro noise model in a Kalman filter by suitably augmenting the state vector has the potential to provide estimates of the sensor bias when the observability requirement is satisfied. Early implementations of gyro noise models in Kalman filters can be found in [17].

An estimate of the attitude would imply the derivation of the dynamics of the robot, which we wish to avoid for the reasons listed in the previous section. In order to do so we relate the gyro output signal to the bias and the angular velocity of the vehicle using the simple and realistic model [8]. In this model the angular velocity about a particular axis $\omega = \dot{\theta}$ is related to the gyro output ω_m according to the equation:

$$\dot{\theta} = \omega_m + b + n_r \tag{3}$$

where b is the drift-rate bias and n_r is the drift-rate noise. n_r is assumed to be a Gaussian white-noise process with covariance N_r. The drift-rate bias b is not a static quantity but is driven by a second Gaussian white-noise process, the gyro drift-rate ramp noise n_w. Thus $\dot{b} = n_w$ with covariance N_w. The two noise processes are assumed to be uncorrelated.

4 3-D Attitude Estimation

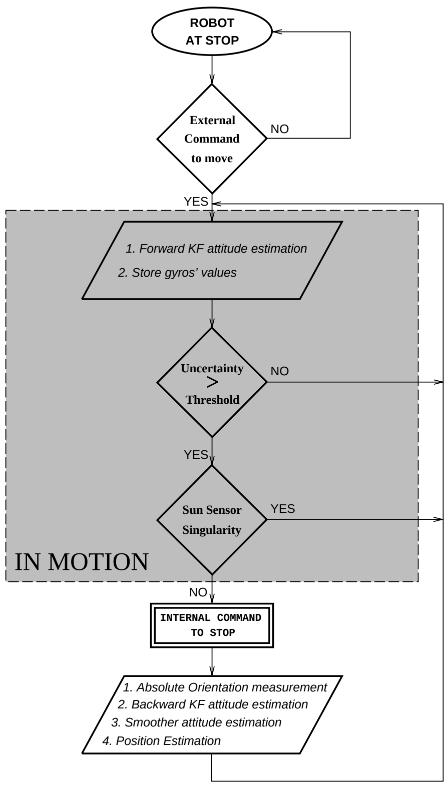

Figure 1: Algorithm Flow Chart: While the robot is in motion the forward Kalman filter uses gyro data to produce (in real-time) a first approximation of the attitude estimate. When the covariance of this estimate exceeds a preset threshold the robot is stopped. An absolute orientation measurement is made using the sun sensor and the three-axis accelerometer. A backward estimate is computed (off-line) and its results are combined (off-line) with the estimate from the forward filter using a smoother. Finally, the position is estimated (off-line) using the (smoothed) attitude estimate for each instant of the trajectory.

The proposed method in the 3D case is summarized in Figure 1. It should be noted that only the forward filter estimate is available in real-time. The smoother runs off-line (during the times that the robot is halted). This technique is not limited to robots used for Mars exploration. It can be applied to any other autonomous vehicle equipped an equivalent set of sensors. The mixing of high frequency inertial sensors with low frequency absolute (position or orientation) sensors is becoming common in mobile robotics. Robots equipped with GPS or landmark tracking devices, usually carry additional sensors that can be used for localization when the GPS signal degrades or the landmarks are occluded. Our framework could be used to combine the data from such sensor sets as well.

4.1 Attitude kinematics

We use quaternions to parameterize the robot's attitude for three practical reasons. First, the prediction equations are treated linearly, secondly the representation is free of singularities and finally the attitude matrix is algebraic in the quaternion components, thus eliminating the need for transcendental functions. The reader is referred to [5] for a review on quaternions.

The physical counterparts of quaternions are the rotational axis $\hat{n}$ and the rotational angle θ that are used in the Euler theorem regarding finite rotations. By taking the vector part of a quaternion and normalizing it, we can find the rotational axis, and from the last parameter we can obtain the angle of rotation [7]. Following the notation in [13], a unit quaternion is defined as:

$$q = [q_1 \quad q_2 \quad q_3 \quad q_4]^T \qquad q^T\, q = 1 \tag{4}$$

where the first three elements of the quaternion can be written in a compact form as:

$$\vec{q} = \hat{n} sin(\theta/2) \tag{5}$$

The attitude matrix is obtained from the quaternion according to the relation:

$$A(q) = (|q_4|^2 - |\vec{q}|^2) I_{3\times3} + 2\vec{q}\vec{q}^T + 2q_4 \left[\left[\ \vec{q}\ \right]\right] \tag{6}$$

where

$$\left[\left[\ \vec{q}\ \right]\right] = \begin{bmatrix} 0 & q_3 & -q_2 \\ -q_3 & 0 & q_1 \\ q_2 & -q_1 & 0 \end{bmatrix} \tag{7}$$

is a 3×3 skew symmetric matrix generated by the 3×1 vector $\vec{q}$. The matrix $A(q)$ transforms representations of vectors in the reference coordinate system to representations in the body fixed coordinate system. The rate of change of the attitude matrix with time is given by:

$$\frac{d}{dt} A(t) = \left[\left[\ \vec{\omega}(t)\ \right]\right] A(t) \tag{8}$$

where the corresponding rate for the quaternion is:

$$\frac{d}{dt} q(t) = \frac{1}{2} \Omega(\vec{\omega}(t)) q(t) \tag{9}$$

with

$$\Omega(\vec{\omega}) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \tag{10}$$

At this point we present an approximate body-referenced representation of the error state vector and covariance matrix. The error state includes the bias error and the quaternion error. The bias error is defined as the difference between the true and estimated bias.

$$\Delta\vec{b} = \vec{b}_{true} - \vec{b}_i \tag{11}$$

The quaternion error is not the arithmetic difference between the true and estimated but it is expressed as the quaternion which must be composed with the estimated quaternion in order to obtain the true quaternion.

$$\delta q = q_{true} \otimes q_i^{-1} \quad or \quad q_{true} = \delta q \otimes q_i \tag{12}$$

The advantage of this representation is that since the incremental quaternion corresponds very closely to a small rotation, the fourth component will be close to unity and thus the attitude information of interest is contained in the three vector component $\delta\vec{q}$ where

$$\delta q \simeq [\delta\vec{q} \ \ 1]^T \tag{13}$$

Starting from equations:

$$\frac{d}{dt} q_{true} = \frac{1}{2}\Omega(\dot{\vec{\theta}}_{true}) q_{true} \tag{14}$$

and

$$\frac{d}{dt} q_i = \frac{1}{2}\Omega(\dot{\vec{\theta}}_i) q_i \tag{15}$$

where $\dot{\vec{\theta}}_{true}$ is the true rate of change of the attitude and $\dot{\vec{\theta}}_i$ is estimated from the measurements provided by the gyros, it can be shown [19] that

$$\frac{d}{dt}\delta\vec{q} = [[\ \vec{\omega}_m\ ]]\,\delta\vec{q} - \frac{1}{2}(\Delta\vec{b} + \vec{n}_r) \quad \frac{d}{dt}\delta q_4 = 0 \tag{16}$$

where $\vec{\omega}_m$ is the output of the gyros. Using the infinitesimal angle assumption in Equation 5, $\delta\vec{q}$ can be written as $\delta\vec{q} = \frac{1}{2}\delta\vec{\theta}$. Thus Equation 16 can be rewritten as:

$$\frac{d}{dt}\delta\vec{\theta} = [[\ \vec{\omega}_m\ ]]\,\delta\vec{\theta} - (\Delta\vec{b} + \vec{n}_r) \tag{17}$$

Differentiating Equation 11 and assuming $\dot{\vec{b}}_{true} = \vec{n}_w$ and $\dot{\vec{b}}_i = 0$, the bias error dynamic equation is $\frac{d}{dt}\Delta\vec{b} = \vec{n}_w$ which when combined with Equation 17 yields the error state equation:

$$\frac{d}{dt}\begin{bmatrix} \delta\vec{\theta} \\ \Delta\vec{b} \end{bmatrix} = \begin{bmatrix} [[\ \vec{\omega}_m\ ]] & -I_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}\begin{bmatrix} \delta\vec{\theta} \\ \Delta\vec{b} \end{bmatrix} + \begin{bmatrix} -I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} \end{bmatrix}\begin{bmatrix} \vec{n}_r \\ \vec{n}_w \end{bmatrix} \tag{18}$$

In a more compact form Equation 18 is:

$$\frac{d}{dt}\Delta x = F\Delta x + Gn \tag{19}$$

4.2 Discrete system: Indirect forward Kalman filter equations

4.2.1 Propagation

At this point we define $q_{k/k}$ ($\vec{b}_{k/k}$) as the quaternion (bias) estimate at time t_k based on data up to and including $z(t_k)$, $q_{k/k-1}$ ($\vec{b}_{k/k-1}$) the quaternion (bias) estimate at time time t_{k-1} propagated to t_k, right before the measurement update at t_k. The estimated angular velocity is defined (before and after the update) as:

$$\vec{\omega}_{k/k-1} = \vec{\omega}_m(t_k) - \vec{b}_{k/k-1} \quad \vec{\omega}_{k/k} = \vec{\omega}_m(t_k) - \vec{b}_{k/k} \tag{20}$$

Following [25], the full estimated quaternion is propagated over the interval $\Delta t_k = t_k - t_{k-1}$ as follows:

$$q_{k/k-1} = \left\{ \begin{array}{c} exp[\frac{1}{2}\Omega(\vec{\omega}_{avg})\Delta t_k] + [\Omega(\vec{\omega}_{k/k-1})\Omega(\vec{\omega}_{k-1/k-1}) \\ -\Omega(\vec{\omega}_{k-1/k-1})\Omega(\vec{\omega}_{k/k-1})]\Delta t_k^2/48 \end{array} \right\} q_{k-1/k-1}$$

where the average angular velocity for this interval is approximately

$$\vec{\omega}_{avg} = \frac{\vec{\omega}_{k/k-1} + \vec{\omega}_{k-1/k-1}}{2} \tag{21}$$

The bias estimate is constant over the propagation interval $\vec{b}_{k/k-1} = \vec{b}_{k-1/k-1}$. The propagation equation for the error state covariance is

$$P_{k/k-1} = \Phi(k,k-1)P_{k-1/k-1}\Phi^T(k,k-1) + Q_k \tag{22}$$

If the average angular velocity $\vec{\omega}_{avg}$ is constant over the interval Δt_k, with magnitude ω_{avg} then the discrete system transition matrix $\Phi(k,k-1)$ can be calculated from Equation 18 as:

$$\Phi(k,k-1) = \begin{bmatrix} exp([[\ \vec{\omega}_{avg}\ ]]\,\Delta t_k) & -\Psi(\Delta t_k) \\ 0_{3\times3} & I_{3\times3} \end{bmatrix} \tag{23}$$

where the matrix Ψ is

$$\begin{aligned} \Psi(\Delta t_k) = I_{3\times3}\Delta t_k &+ [[\ \vec{\omega}_{avg}\ ]]\,(1 - \cos(\omega_{avg}\Delta t_k)/\omega_{avg}^2 \\ &+ [[\ \vec{\omega}_{avg}\ ]]^2\,(\omega_{avg}\Delta t_k - \sin(\omega_{avg}\Delta t_k))/\omega_{avg}^3 \end{aligned} \tag{24}$$

4.2.2 Update

When the rover stops, an absolute orientation measurement is available from the sun sensor and the accelerometer. This is used to update the estimated error state and the covariance [15]. The Kalman gain matrix is given by:

$$K_k = P_{k/k-1}H_k^T\ (H_k P_{k/k-1} H_k^T + R_k)^{-1} \tag{25}$$

The updated covariance and error state equations are:

$$P_{k/k} = P_{k/k-1} - K_k H_k P_{k/k-1} \quad (26)$$

$$\Delta x_{k/k} = \Delta x_{k/k-1} + K_k \Delta z(t_k) \quad (27)$$

or

$$\begin{bmatrix} \delta\vec{\theta}_{k/k} & \Delta\vec{b}_{k/k} \end{bmatrix}^T = K_k \Delta z(t_k) \quad (28)$$

where $\Delta z(t_k)$ is the measurement residual. The propagated error $\Delta x_{k/k-1}$ is zero because we have implemented the feedback formulation of the Indirect Kalman filter. Every time we have a measurement the update is included in the full state and thus the next estimate of the error state $\Delta x_{k/k-1}$ is assumed to be zero. This update is:

$$q_{k/k} = \delta q_{k/k} \otimes q_{k/k-1} = \begin{bmatrix} \delta\vec{q}_{k/k} & 1 \end{bmatrix}^T \otimes q_{k/k-1} \quad (29)$$

where

$$\delta\vec{q}_{k/k} = (1/2)\delta\vec{\theta}_{k/k} \quad \vec{b}_{k/k} = \vec{b}_{k/k-1} + \Delta\vec{b}_{k/k} \quad (30)$$

4.2.3 Observation model

The attitude sensors considered here are the sun sensor and the (three-axial) accelerometer both used when the rover is at stop. The first one measures the unit vector towards the sun and the second one provides the unit vector in the vertical direction. These measurements depend explicitly on the attitude but not on the gyroscopes' biases. Let the observed vector in the sensor frame be

$$\hat{p}_S(t_k) = T_{S\leftarrow B} A(q(t_k))\hat{p}_I + \vec{n}_p \quad (31)$$

where $\hat{p}_S$ is the measured unit vector in the sensor frame, $\hat{p}_I$ is the reference vector in the inertial frame (pointing towards the direction of the sun for example - we assume that it does not change significantly for small time intervals), $T_{S\leftarrow B}$ is the transformation matrix from the body frame to the sensor frame, $A(q(tk))$ is the true orientation matrix of the robot at time t_k, $q(t_k)$ is the true attitude quaternion (q_{true}) at time t_k and $\vec{n}_p$ is the sensor noise. If $\hat{p}_{S,k/k-1}$ is the measured unit vector estimate at time t_k and Π is the matrix that projects the measurements on a plane perpendicular to the sensor boresight, then the residual $\Delta z(t_k)$ can be written as:

$$\Delta z(t_k) = \Pi\ (\hat{p}_S(t_k) - \hat{p}_{S,k/k-1})$$
$$= \Pi\ (T_{S\leftarrow B} A(q(t_k))\hat{p}_I + \vec{n}_p) - T_{S\leftarrow B} A(q_{k-1/k})\hat{p}_I) \quad (32)$$

The true orientation matrix is

$$A(q(t_k)) = A(\delta\vec{q}(t_k) \otimes q_{k/k-1}) = A(\delta\vec{q}(t_k))A(q_{k/k-1}) \quad (33)$$

Using Equation 6 and making small rotation angle approximations we can write

$$A(\delta\vec{q}(t_k)) \simeq I_{3\times 3} + 2\,[[\ \delta\vec{q}(t_k)\ ]] \quad (34)$$

Substituting back in Equation 33 for $A(\delta\vec{q}(t_k))$ and using the resultant expression for $A(q(t_k))$ in Equation 32 we get similar results to those in [22]:

$$\Delta z(t_k) = \Pi\ (T_{S\leftarrow B}\,[[\ \delta\vec{\theta}(t_k)\ ]]\,A(q_{k/k-1}) + \vec{n}_p) =$$
$$\begin{bmatrix} \hat{s}_x \times A(q_{k/k-1})\hat{p}_I \\ \hat{s}_y \times A(q_{k/k-1})\hat{p}_I \end{bmatrix} \delta\vec{\theta}(t_k) + \Pi\ \vec{n}_p =$$
$$h_{k/k-1}\delta\vec{\theta}(t_k) + \Pi\ \vec{n}_p \quad (35)$$

where $\hat{s}_x$ and $\hat{s}_y$ are the unit vectors along the sensor axes. From the previous equation it is obvious that the observation model is non-linear and to calculate the sensitivity (measurement) matrix we have to derive the partial derivatives of Δz with respect to the estimated error states. The measurements are independent from the gyro biases and thus:

$$H_k = \frac{\partial(\Delta z)}{\partial(\delta\vec{\theta})} = \begin{bmatrix} h_{k/k-1} & 0_{2\times 3} \end{bmatrix} \quad (36)$$

4.3 Backward filter

In the flow chart shown in Figure 1 we see that the robot stops every time the uncertainty grows over a preset threshold. Then the backward filter is engaged and the last attitude estimate is propagated back in time. This last estimate is very precise because it is heavily based on the absolute orientation measurements acquired when the robot stopped. While the backward filter is close to its starting point it is able to provide estimates of higher confidence than those of the forward filter. In order to derive the equations for the backward Indirect Kalman filter we start from the equations of the system for the forward case:

$$\dot{x} = Fx + Gw \quad and \quad z = Hx + v \quad (37)$$

By defining $\tau = T - t$, where τ is the backward time variable and $T = t_k - t_{k-M}$ is the time interval of smoothing, the backward system equation can be derived from:

$$\frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = -\dot{x} \quad \frac{dx_b}{d\tau} = -Fx_b - Gw \quad (38)$$

Making the appropriate substitutions we get the following equation for the quaternion estimate propagation:

$$q_{b,k-1/k-1} = \left\{ \begin{matrix} exp[\frac{1}{2}\Omega(\vec{\omega}_{avg})\Delta t_k] + [\Omega(\vec{\omega}_{k/k-1})\Omega(\vec{\omega}_{k-1/k-1}) - \\ \Omega(\vec{\omega}_{k-1/k-1})\Omega(\vec{\omega}_{k/k-1})]\Delta t_k^2/48 \end{matrix} \right\}^T q_{b,k/k-1}$$

The bias propagation remains the same as before since the direction of propagation does not affect an assumed constant variable. The backward propagation equation for the covariance is now:

$$P_{b,k-1/k-1} = \Phi_b(k-1,k)P_{b,k/k-1}\Phi_b^T(k-1,k) + Q_{b,k} \quad (39)$$

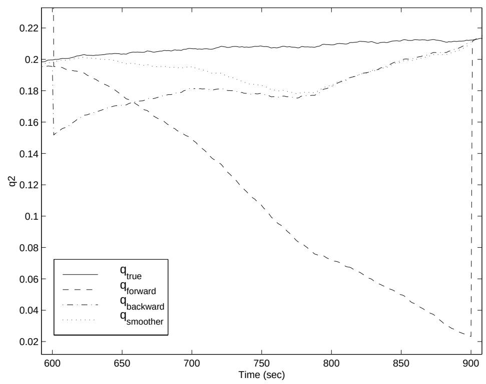

Figure 2: This is the usual outcome due to the bias estimation. The forward filter estimate drifts to the right because it has underestimated the gyro bias. The backward filter overestimates and thus drifts to the left (in the opposite direction). The smoothed estimate outperforms both filters minimizing the average estimation error.

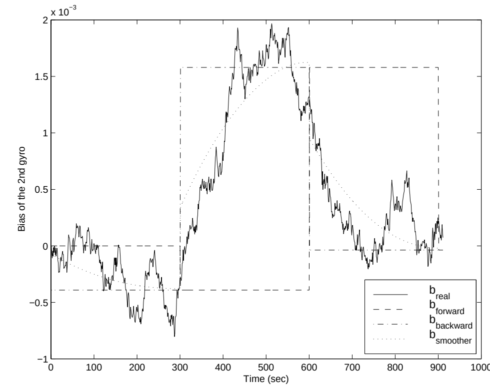

Figure 3: For the second gyro, we show the true bias value, the forward filter's estimate, the backward filter's estimate and the smoothed estimate of the bias. The smoothed (total) estimate stays close to the backward filter estimate for the second half of each smoothing interval while for the first part it depends on both the forward filter's estimate and the backward filter's estimate. This is due to the fact that the initial bias value for the backward filter is more trustworthy for this time interval than the initial value of the forward filter. The asymmetry is due to the fact that the backward filter works with an "initial" estimate which is actually computed after the motion.

No new absolute measurements are collected during the backward propagation of the filter and thus, the update equations and the observation model for the backward filter are not considered.

4.4 Smoother

The smoother constructs the best estimate of the state of the system over a time period using all the measurements in that time interval [18]. In our case, the time for which the robot stops to get an absolute orientation measurement allows for post-processing and therefore application of the smoother. In order to calculate the total (smoothed) estimate we use the following equation[1]:

$$P_{total}^{-1}\hat{x}_{total} = P_f^{-1}\hat{x}_f + P_b^{-1}\hat{x}_b \tag{40}$$

Each covariance matrix P_f, P_b and P_{total} represents the uncertainty of the corresponding estimate. The higher the uncertainty, the larger the covariance matrix. Equation 40 weighs each of the available estimates (from the forward and the backward filter) according to their certainty. The result is the optimal estimate possible, if all the measurements of the time interval of smoothing were available at once. The significant improvement in the quality of the 3D estimate is shown in Figure 4. Different estimated quantities calculated in a representative trial are depicted in Figures 2 and 3.

[1] Applying this in 3D is somewhat involved because of the particular form of the error quaternion used. The interested reader is referred to [19] for the technical details

5 From Attitude Estimates to Position Estimates

The accuracy of the position estimate depends heavily on the accuracy of the attitude estimate. Though the position can be calculated in real-time using the output of the forward Kalman filter we choose not to do that. Instead in our algorithm the position estimation takes place off-line as described in Figure 1. After the vehicle stops to collect an absolute orientation measurement the off-line smoothing of the attitude estimation is performed. The resulting estimate is accurate and is used to compute the current position. As we mentioned before the attitude estimate is an input to Equations 1 and 2. If the integration step is small, we can simplify this calculation as follows. First the increase in position is calculated due to the sensed acceleration and the current velocity:

$$^{L}\Delta p(t_k) = \, ^{L}v(t_k)\,\Delta T + \, ^{L}a(t_k)\,\Delta T^2/2 \tag{41}$$

this increment is then transformed to global coordinates using $^{G}\Delta p(t_k) = \, ^{G}_{L}A(q(t_k))\, ^{L}\Delta p(t_k)$, before it can be used to compute the next position using

$$^{G}p(t_{k+1}) = \, ^{G}p(t_k) + \, ^{G}\Delta p(t_k) \tag{42}$$

The velocity increment during every measurement cycle is $^{L}\Delta v(t_k) = \, ^{L}a(t_k)\,\Delta T$. In global coordinates, we

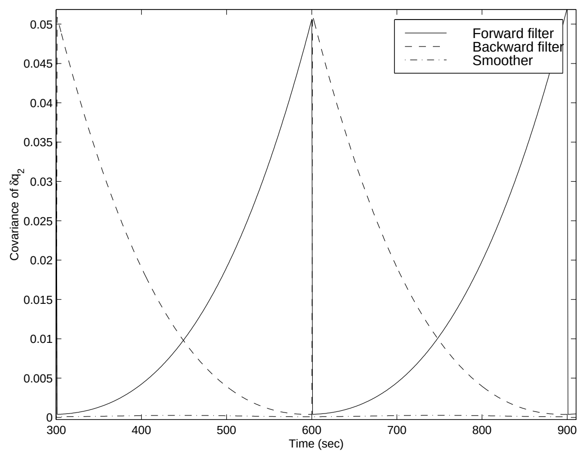

Figure 4: The covariance related to q_2 from the forward filter, backward filter and smoother is shown. At all times the total covariance is lower than either of the corresponding ones calculated from the two filters. Its value remains bounded and varies slightly during the smoothing interval.

have ${}^G\,\Delta v(t_k) = {}^G_L A(q(t_k))\ {}^L\,\Delta v(t_k)$. The new velocity is

$$^G\ v(t_{k+1}) = \ ^G\ v(t_k) + \ ^G\ \Delta v(t_{k+1}) \tag{43}$$

This result has to be transformed to local coordinates before it is fed back for the next position update:

$$^L\ v(t_{k+1}) = {}^L_G A(q(t_{k+1}))\ ^G\ v(t_{k+1}) \tag{44}$$

6 Conclusion

In this paper we decomposed the localization algorithm into attitude estimation and, subsequently, position estimation. A novel approach that incorporates a smoother was presented. An Indirect (error-state) Kalman filter that incorporates inertial navigation and absolute measurements was developed for this purpose. The dynamic model was replaced by gyro modeling which relies on the integration of the kinematic equations. The error state equations for the three dimensional case were derived and used to formulate the filter's time-variant system model and non-linear observation model. Quaternions were selected for the three dimensional attitude representation. Finally, the improvement due to the proposed method was demonstrated in simulation. Uniformly smaller values of the covariance of the estimate were sustained throughout each of the trials. It should be noted that due to the lack of vehicle specific dynamic modeling the proposed approach is general and may be used on any vehicle chassis with an equivalent set of sensors. Future directions of research include applications (extensions) of this method to cases where the INS sensors are fused with other absolute sensors that measure position (e.g. vision cues, star sensors, beacons etc.)

Acknowledgments

The authors would like to thank Dr. D. Bayard for several illuminating discussions. We also thank Dr. B. Balaram and Dr. G. Rodriguez for technical discussions.

References

[1] B. Barshan and H. F. Durrant-Whyte. Inertial navigation systems for mobile robots. *IEEE Transactions on Robotics and Automation*, 11(3):328–342, June 1995.

[2] Ph. Bonnifait and G. Garcia. A multisensor localization algorithm for mobile robots and it's real-time experimental validation. In *Proceedings of the 1996 IEEE International Conference on Robotics and Automation*, pages 1395–1400, 1996.

[3] J. Borenstein and L. Feng. Correction of systematic odometry errors in mobile robots. In *Proceedings of the 1995 IEEE International Conference on Robotics and Automation*, pages 569–574, 1995.

[4] J. Borenstein and L. Feng. Gyrodometry: A new method for combining data from gyros and odometry in mobile robots. In *Proceedings of the 1996 IEEE International Conference on Robotics and Automation*, pages 423–428, 1996.

[5] J.C.K. Chou. Quaternion kinematic and dynamic differential equations. *IEEE Transactions on Robotics and Automation*, 8(1):53–64, Feb. 1992.

[6] S. Cooper and H. F. Durrant-Whyte. A frequency response method for multi-sensor high-speed navigation systems. In *Proceedings of the 1994 IEEE Conference on Multisensor Fusion and Integration Systems*, 1994.

[7] J.J. Craig. *Introduction to Robotics*, chapter 2, pages 55–56. Addison-Wesley, 2nd edition, 1989.

[8] R.L. Farrenkopf. Analytic steady-state accuracy solutions for two common spacecraft estimators. *Journal of Guidance and Control*, 1:282–284, July-Aug. 1978.

[9] Y. Fuke and E. Krotkov. Dead reckoning for a lunar rover on uneven terrain. In *Proceedings of the 1996 IEEE International Conference on Robotics and Automation*, pages 411–416, 1996.

[10] M. S. Grewal and A. P. Andrews. *Kalman Filtering, Theory and Practice*. Prentice Hall, 1993.

[11] S. Hayati, R. Volpe, P. Backes, J. Balaram, and R. Welch. Microrover research for exploration of mars. In *Proceedings of the 1996 AIAA Forum on Advanced Developments in Space Robotics*, University of Wisconsin, Madison, August 1-2 1996.

[12] E.J. Lefferts and F.L. Markley. Dynamics modeling for attitude determination. *AIAA Paper 76-1910*, Aug. 1976.

[13] E.J. Lefferts, F.L. Markley, and M.D. Shuster. Kalman filtering for spacecraft attitude estimation. *Journal of Guidance, Control, and Dynamics*, 5(5):417–429, Sept.-Oct. 1982.

[14] J. J. Leonard and H. F. Durrant-Whyte. Mobile robot localization by tracking geometric beacons. *IEEE Transactions on Robotics and Automation*, 7(3):376–382, June 1991.

[15] J. M. Mendel. *Lessons in Digital Estimation Theory*, chapter 17. Prentice-Hall, 1987.

[16] G.C. Newton. Inertial guidance limitations imposed by fluctuation phenomena in gyroscopes. In *Proceedings of the IRE*, volume 48, pages 520–527, April 1960.

[17] J.E. Potter and W.E. Vander Velde. Optimum mixing of gyroscope and star tracker data. *Journal of Spacecraft and Rockets*, 5:536–540, May 1968.

[18] S.R. Reynolds. Fixed interval smoothing: Revisited. *Journal of Guidance*, 13(5), Sep.-Oct. 1990.

[19] S. I. Roumeliotis, G. S. Sukhatme, and G. A. Bekey. Smoother-based 3-d attitude estimation for mobile robot localization. Technical Report IRIS-98-363, USC, 1998. http://iris.usc.edu/ irislib/.

[20] S. I. Roumeliotis, G. S. Sukhatme, and G. A. Bekey. Circumventing dynamic modeling: Evaluation of the error-state kalman filter applied to mobile robot localization. In *Proceedings of the 1999 IEEE International Conference on Robotics and Automation*, 1999.

[21] R.E. Scheid, D.S. Bayard, J.(Bob) Balaram, and D.B. Gennery. On-board state estimation for planetary aerobots. In *12th Lighter-Than-Air Systems Technology Conference, AIAA International Balloon Technology Conference, 14th Aerodynamic Decelerator Systems Technology Conference and Seminar*, San Francisco, CA, June 3-5 1997. AIAA.

[22] J. Sedlak and D. Chu. Kalman filter estimation of attitude and gyro bias with the quest observation model". *Advances in Astronautical Sciences*, 84(1):683–696, 1993.

[23] S. Thrun. Bayesian landmark learning for mobile robot localization. *Machine Learning*, 1998. to appear.

[24] J. Vaganay, M.J. Aldon, and A. Fournier. Mobile robot attitude estimation by fusion of inertial data. In *Proceedings of the 1993 IEEE International Conference on Robotics and Automation*, pages 277–282, 1993.

[25] J.R. Wertz, editor. *Spacecraft Attitude Determination and Control*, volume 73 of *Astrophysics and Space Science Library*. D. Reidel Publishing Company, Dordrecht, The Netherlands, 1978.